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Appropriate Use of Box-Cox Transforms for Seasonal Adjustment

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Analytical Services Branch

Methodology Advisory Committee

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APPROPRIATE USE OF BOX–COX TRANSFORMS FOR SEASONAL ADJUSTMENT

Leanne Chhay and Alex Stuckey
Analytical Services Branch

QUESTIONS FOR THE COMMITTEE

1. Are the methods of selecting a transform presented in this paper appropriate to the topic of interest?
2. Are the quality assessment measures appropriate to the topic of interest?
3. Is the Box–Cox transform appropriate for seasonal adjustment when the estimated Box–Cox parameter falls out of the interval $[0,1]$?
4. Are there any suggestions as to how the naïve estimators of the seasonally adjusted series can be improved?

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APPROPRIATE USE OF BOX–COX TRANSFORMS FOR SEASONAL ADJUSTMENT

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ABSTRACT

It is well established that appropriate Box–Cox transformation of data is in some cases desirable when using standard analytical techniques. For example, the use of such transforms for variance stabilization in regression and ARIMA time series modelling were developed in Box and Cox (1964) and Box and Jenkins (1970) respectively.

In the setting of seasonal adjustment of time series such transforms offer a compromise between, and extension beyond, the standard additive and multiplicative options of decomposition models. In particular, an appropriate transformation may lead to more stable seasonal factor estimates and in turn reduce current end revisions to seasonally adjusted estimates obtained using the ABS X11-based concurrent method.

The empirical study presented here evaluates two existing methods of selecting the Box–Cox parameter, and proposes two new methods for the purposes of seasonal adjustment. These methods of transformation selection are compared to an optimal transform found by a simple search method. Quality is assessed via measures relating to the volatility of, and current end revisions to, the resulting seasonally adjusted and trend series.

The existing methods evaluated are a maximum likelihood approach, given a seasonal ARIMA model (Hipel *et al.*, 1977), and a time series variance stabilisation method (Guerrero, 1993). A simple alternative is trialled that uses appropriate seasonal dummy variables in a regression ARIMA model. The aim of this latter approach is to apply a transform that results in stable additive seasonal factors. An additional method optimises the Box-Cox parameter with respect to a quality indicator developed by Statistics Canada, known as the $M7$ value, which provides a measure of the reliability of the seasonal adjustment.

1. INTRODUCTION

At present, the Australian Bureau of Statistics (ABS) is investigating the use of Box–Cox transforms in improving seasonal adjustment of official time series. Box–Cox functions are a broad family of power transforms, encompassing also the standard additive and multiplicative options that the ABS currently use for the decomposition of a time series. The transform is described by the function

$$O_t^{(\lambda)} = \begin{cases} \dfrac{O_t^{\lambda} - 1}{\lambda} & , \lambda \neq 0 \\ \log(O_t) & , \lambda = 0 \end{cases}$$

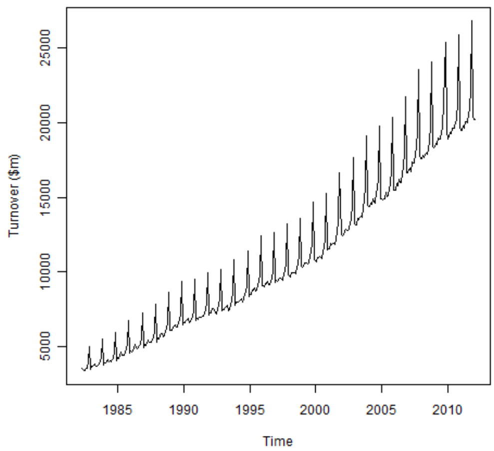
One of the earliest case studies in relation to the use of Box–Cox transforms for seasonal adjustment was that of engineering sales, conducted by Chatfield and Prothero in 1973. This case study had generated much discussion in regards to the appropriate use of the multiplicative model for that particular time series. Research following from this case study showed that the multiplicative option was over-transforming the series, and identified a particular Box–Cox transform to be a better alternative, in terms of achieving more stable, additive seasonality in the transformed series.

Additionally, Proietti and Riani (2007) pointed out that Box–Cox transforms could be relevant to series where seasonality is the most prominent source of variation, suggesting sales, tourism and industrial production type data as potential candidates for Box–Cox transformation.

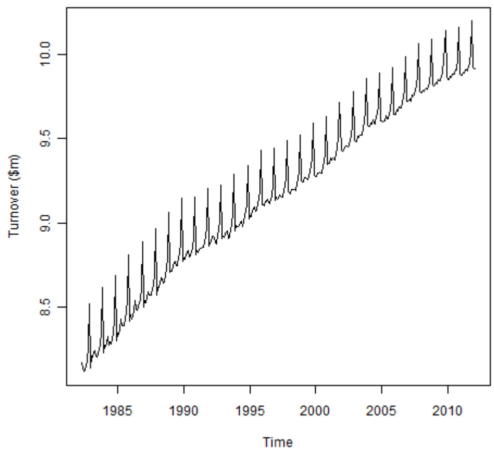
Using the ABS Retail Trade series as an example, we are interested in whether or not Box–Cox transforms can lead to improved seasonal factors. From the plots below (see figure 1.1), which illustrate the original series, the multiplicative adjustment and the Box–Cox transformation (with parameter $\lambda = 0.2$) respectively, we can see the potential for using Box–Cox transforms to achieve stable, additive seasonality. The original data show that the December peaks are growing in magnitude with time, whereas the multiplicative adjustment (corresponding to the logarithmic transform) shows a decline in seasonality. The Box–Cox transform on the other hand, appears to produce constant seasonality relative to that of the previous two decomposition models. This is one motivating example illustrating the potential advantage of using Box–Cox transforms for seasonal adjustment.

1.1 Retail Trade, Australia

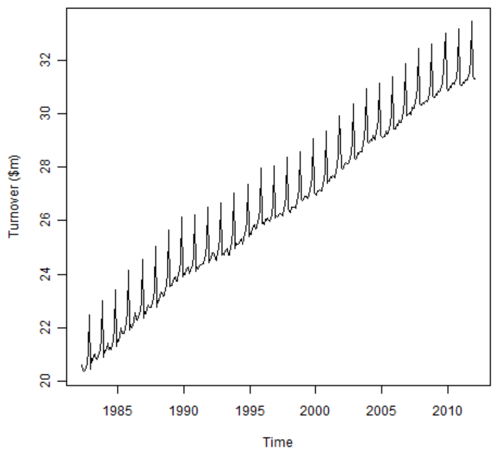
(a) Original series (no transform)



(b) Logarithmic transform ($\lambda = 0$)



(c) Box–Cox transform ($\lambda = 0.2$)



Since the formulation of the power transformation by Box and Cox (1964), there has been an extensive amount of research into the estimation of the Box–Cox parameter, λ. Much emphasis has been placed on the use of these transformations in obtaining adequate autoregressive integrated moving average (ARIMA) models. Guerrero (1993) states that for a time series decomposition, unstable seasonal components are commonly associated with a series that has non-constant variance. This forms another driver for this study, as data transformation has been known to correct for heteroscedastic and non-normal residuals. The set of ABS Short Term Overseas Arrivals and Departures series, most being naturally volatile, present themselves as a possible candidate for implementing Box–Cox transformations. We will further investigate this in a simulation study by looking at series with varying degrees of volatility.

With the above drivers in mind, the main aim of this paper is to examine various methods put forth in literature regarding the selection of λ, and to compare their performances against quality measures such as revisions to seasonally adjusted and trend data, volatility indicators and tests for stable seasonality as well as residual seasonality.

This paper will begin with a literature review of the various selection methods for the estimation of the Box–Cox parameter, as well as issues that potentially affect the seasonal adjustment process. We then present our empirical study in which we assess the performance of these estimators. Concluding remarks are given in the last section, as well as issues to consider for future implementation.

2. LITERATURE REVIEW

To date, the use of Box–Cox transforms in the production of official time series statistics coupled with the filter-based X11 methodology (Shiskin *et al.*, 1967) is not common practice. There is ongoing discussion in regard to issues arising in its practical implementation for seasonal adjustment. Research in this area tackles issues such as the choice of transform parameter λ, bias reduction in inverse transformations, improvements to forecasting and seasonality and trend estimation. Official statistical agencies may also be interested in approaches to handling seasonal balance constraints, identification of outliers on the transformed scales, aggregation and consistency.

The concept of applying Box–Cox transforms to enhance time series analysis is not new. In Box and Cox (1964), the power transform was discussed in the context of analysis of variance and multiple regression. Box and Jenkins (1970) broadened the application of these transforms to time series analysis, suggesting that such transforms aided in obtaining adequate autoregressive integrated moving average (ARIMA) models, that is, ARIMA models with valid assumptions of homoscedastic and normal residuals. Both these papers proposed the use of maximum likelihood in estimating λ, upon initial specification of an ARIMA model. Given such a model, λ can then be estimated simultaneously with the parameters of the specified ARIMA model (Hipel *et al.*, 1977).

An algorithm developed by Ansley *et al.* (1977) conducts a grid search of various values for λ, however requiring a preliminary estimate for initialisation. This approach has been known to be sensitive to the autocorrelation structure of the series, so the estimation of the ARIMA model could potentially change during the grid search (Guerrero, 1993; Granger and Newbold, 1976).

Jenkins (1979) suggested the use of 'range-mean' plots in choosing a preliminary, crude estimate of the Box–Cox parameter. Unlike those discussed above, this method is model-independent, requiring no initial specification of an ARIMA model. The range is plotted as a function of the mean, providing an indication of whether a transformation is needed. The author clearly stated that the main use of these plots is to distinguish between the logarithmic, square-root and identity transformations, as opposed to obtaining an exact value of λ.

It is well established in literature that Box–Cox transforms can correct for heteroscedastic and non-normal errors. In obtaining homoscedasticity, Guerrero (1993) proposed a method of selecting λ by minimising the coefficient of variation. As an extended analysis of the engineering sales dataset first investigated by Chatfield and Prothero (1973), Guerrero's proposed method was visually found to yield a more stable seasonal pattern, hence, an improvement in seasonal adjustment.

Authors Shulman and McKenzie (1984) from the U.S. Bureau of the Census conducted a case study on the use of Box–Cox transforms, using maximum likelihood estimation based on a particular seasonal ARIMA model and the X11 method. Numerical quality measures were employed to assess the performance of the seasonally adjusted series, for which the authors concluded that the choice of λ was rather sensitive to particular span of the series used. Using transforms of this kind the authors achieved reductions to revisions in two of the series tested.

In a more recent paper, Proietti and Riani (2007) proposed the use of a forward search approach to obtain a robust estimate λ. The authors imposed the basic structural model in estimating the seasonally adjusted series, providing analytical solutions for their conditional mean and conditional variance on the original scale. Their paper highlighted another important issue, that of bias being induced at the stage of reverse-transformation. Using the same dataset that Chatfield and Prothero (1973) had, the authors reported accuracy measures such as mean error and mean square error for their proposed correction factors, derived using numerical integration, along with approximate corrections proposed by Taylor (1986) and Guerrero (1993). Their results illustrate that the method of numerical integration is the most accurate, providing the fastest and most reliable method of computing the required conditional variances. De Livera *et al.* (2011) propose a maximum likelihood approach to estimating λ simultaneously with smoothing parameters used for seasonality and trend estimation and coefficients of an ARMA error specification.

In the context of non-parametric methods however, those such as X11 and X12–ARIMA, no alternatives to the naïve seasonally adjusted have been suggested by Proietti and Riani (2007). Thomson and Ozaki (2002) also considered the bias effects of reversing a simple power transform, proposing ad hoc bias corrections for the individual trend, seasonal and irregular components. Studies on simulated and official economic time series were conducted using SABL (Cleveland *et al.*, 1978) to estimate the individual components, with results showing a marked improvement over the uncorrected series, in the case of strong seasonality. Much emphasis has been placed on the computation of the trend series, with the authors claiming, based on the calculated mean trend biases, that the gains from the correction formulae were most substantial in cases where the variation about the trend was greatest.

In the ABS, we are interested in the performance of X11-style seasonal adjustment under Box–Cox transformation in terms of quality of adjustment and revision performance. The main objectives of this paper are to assess the performance of selected and proposed estimators of λ, and to see which one yields the best adjustment based on revisions analysis.

3. METHODS FOR SELECTING THE BOX–COX PARAMETER

Seasonal adjustment is a process whereby systematic and calendar-related effects are estimated and removed from a time series. SEASABS (SEASonal analysis, ABS standards) is the main seasonal adjustment software package developed by and used within the ABS. It has a core processing system based on the filter-based X11 algorithm, as well as X12–ARIMA enhancements, both of which were developed by the U.S. Bureau of the Census. In order to run a concurrent seasonal adjustment, that is, where seasonal factors are estimated at each reference period using available data, a decomposition model must firstly be specified.

A time series O_t, at time t, under the X11 method, can be decomposed into three basic components, namely the trend (T_t), seasonal (S_t) and irregular (I_t). By definition, the seasonally adjusted estimate (SA_t) comprises of the trend and irregular components, where the trend is the underlying level of the original series, and the irregular is what remains after the trend and seasonal components have been removed from the original series. The seasonal component takes into account seasonal influences and calendar-related events, including trading day and moving holiday effects.

In decomposing a time series, the aim is to find a model that yields the most stable seasonal factors. There are two main decomposition models commonly used in the ABS, namely additive and multiplicative. The additive decomposition assumes that the components of the series behave independently of one another:

$$O_t = S_t + T_t + I_t \tag{1}$$

and that the seasonal component remains stable from year to year. An additional constraint is that the seasonal factors are centred around zero. Under a multiplicative decomposition, the trend maintains the same dimensions as the original series, while the seasonal and irregular components are dimensionless factors centred around one. The model is given by

$$O_t = S_t \times T_t \times I_t \tag{2}$$

The multiplicative model is sometimes known as the log-additive model, since it can be written in an additive form by taking the logarithms of (2):

$$\log O_t = \log S_t + \log T_t + \log I_t \tag{3}$$

In fact, both the additive and logarithmic representations belong to a family of power transforms, called the Box–Cox transformation (Box and Cox, 1964):

$$O_t^{(\lambda)} = \begin{cases} \dfrac{O_t^{\lambda} - 1}{\lambda} & , \lambda \neq 0 \\ \log(O_t) & , \lambda = 0 \end{cases} \tag{4}$$

where λ is the Box–Cox parameter. Setting $\lambda = 1$ gives the additive model, which is the identity transform (with a unit shift), whereas $\lambda = 0$ corresponds to the multiplicative model. Generally, there are no constraints to what value λ can take. However, the original data should only contain strictly positive values in order for Box–Cox transformation to be applied.

As outlined in the literature review, there are quite a number of methods available for the selection of λ. The following subsections provide more detail on some of these methods, with a discussion of their implementation into our study. All related coding is done in the `R` environment, using version 2.9.2.

3.1 Maximum likelihood method

In theory, the maximum likelihood method as described by Hipel *et al.* (1977) involves the initial specification of an ARIMA model, where the resulting profile likelihood function is maximised with respect to λ. The Box–Cox parameter that maximises the approximate expression for the log likelihood of all the model parameters over all λ, is given by Hipel *et al.* (1977). That is,

$$\hat{\lambda}_{MLE} = \arg\max_{\lambda} \left\{ \frac{-\frac{N}{2} \ln MSS}{N} + (\lambda - 1) \sum_{t=d+sD+1}^{N} \log(O_t) \right\} \tag{5}$$

where N is the number of observations and MSS is the modified sum of squares (McLeod, 1976). Here, d is the degree of non-seasonal differencing, D is the degree of seasonal differencing, s is the seasonal period and $\hat{D} = d + sD$. The `R` package `FitAR` (McLeod and Zhang, 2008) contains functions that directly carry out the Box–Cox transformation and the above maximisation procedure for a given ARIMA model specification. In this study we use the prevalent monthly airline model $(0\ 1\ 1)(0\ 1\ 1)_{12}$ specification.

3.2 Guerrero's method

Defining X to be a positive random variable with mean $E(X) < \infty$, such that the variance $Var(X)$ can be expressed as a function of the mean, Guerrero (1993) claimed that the value of λ that yields a variance-stabilising transformation must satisfy

$$\frac{\left[Var(X)\right]^{1/2}}{\left[E(X)\right]^{1-\lambda}} = a \ , \ a > 0.$$

In the case of a time series O_t, where it is not possible to estimate dispersion at time t with only one observation, Guerrero suggests dividing the dataset into subsets of size equal to the seasonal period of the series s, truncating either end of the series where necessary. This construction allows a local estimate of the mean and variance for each subset to be calculated, given respectively as

$$\bar{O}_h = \sum_{r=1}^{s} \frac{O_{h,r}}{s} \ ,$$

$$S_h = \sqrt{\sum_{r=1}^{s} \frac{\left(O_{h,r} - \bar{O}_h\right)^2}{s-1}} \ ,$$

where $O_{h,r}$ is the r-th observation of the h-th subset. The problem then reduces to that of minimising the coefficient of variation (CV) of

$$\frac{S_h}{\bar{O}_h^{1-\lambda}}$$

as a function of λ. That is, the estimator for λ using Guerrero's method of variance stabilisation is given by

$$\hat{\lambda}_G = \arg\min_{\lambda} \left\{ CV\left(\frac{S_h}{\bar{O}_h^{1-\lambda}} \right) \right\} . \qquad (6)$$

3.3 Seasonal dummy-variable regression

This method is somewhat similar to the maximum likelihood method in Section 3.1, in that the likelihood function under a specified ARIMA model is computed and then maximised with respect to the Box-Cox parameter. The main difference lies in the specification of the ARIMA model, which is selected using the `auto.arima()` function contained within the `R` package `forecast` v3.04 (Hyndman, 2011). No seasonality orders are specified in the ARIMA model selection, but seasonality is accounted for through the inclusion of seasonal dummies in the regression. The setup of this regression, which imposes additivity in the seasonal factors, aims to provide an adequate linear decomposition for the time series. The seasonality in a model using only dummy variables for each period is constrained to be stable seasonality. The idea behind this method is that the Box-Cox parameter selected will be the parameter that gives a transformed series best able to be modelled by this stable seasonality. Maximising the log likelihood of this regression function, denoted $L(\lambda)$, yields the first proposed estimator

$$\hat{\lambda}_D = \arg\max_{\lambda} \left\{ L(\lambda) \right\} \tag{7}$$

3.4 *M7* optimisation

First developed by Statistics Canada, the $M7$ indicator is a summary measure used to assess the reliability of the seasonal adjustment of a time series. Of the 11 summary measures constructed, this one is particularly scrutinised when assessing the quality of ABS time series. For more details regarding these summary measures, refer to Ladiray and Quenneville (1999, pp. 118-120).

The $M7$ statistic specifically measures the amount of stable seasonality present relative to the amount of moving seasonality. In particular,

$$M7 = \sqrt{\frac{1}{2}\left(\frac{7+3F_M}{F_S}\right)}$$

where F_S and F_M are the statistics obtained from the F-tests carried out for stable seasonality and moving seasonality respectively. Rather than using the $M7$ statistic as a quality measure to compare the performance of these estimators, we will use it as an objective function to minimise with respect to the Box-Cox parameter. This is carried out by reading in the $M7$ value that is output from a diagnostic file as part of the X11 algorithm. In addition, knowing that an original series is clearly seasonal, we are interested in seeing whether seasonality can also be detected on the transformed scale. Thus the $M7$ objective function here will be computed using the transformed time series. The result yields the $M7$-optimising estimator

$$\hat{\lambda}_{M7} = \arg\max_{\lambda} \left(M7 \right) \tag{8}$$

It is also noted here that the $M7$ indicator will not be used as a quality measure for the purposes of this investigation.

3.5 Grid search

In addition to the above selection methods, a simple search of grid values of the Box–Cox parameter will be conducted simultaneously, producing various quality measures for each $\hat{\lambda}$. For the simulation study we will restrict our attention to the range $0 \le \lambda \le 1$ with increments of 0.1. For the study of real data we will expand this range to $-0.5 \le \lambda \le 1.4$ with increments of 0.1.

4. QUALITY MEASURES TO ASSESS SEASONAL ADJUSTMENT UNDER BOX–COX TRANSFORMATION

Revisions to concurrent seasonally adjusted and trend level and movement estimates, the stable residual seasonality test and the $M7$ statistic are three main measures used within the ABS to assess the quality of seasonal adjustment. As one of the proposed estimators employ the $M7$ statistic as an objective function, this study will no longer use it as a performance indicator, however it would still be useful to examine how other estimators perform with respect to it. We will also consider the relative contribution of volatility to growth (RCVG) and the average absolute percentage change (AAPC) as indicators of volatility. For this study, we are particularly interested in the performance of the seasonal adjustment under Box–Cox transformation, compared to the standard additive and multiplicative options the ABS currently uses.

4.1 Revision performance measure

The revision against the benchmark estimate can be interpreted as the size of revision required for an estimate to reach stability. Ideally, revisions to seasonally adjusted and trend data should be kept to a minimum. The introduction of ARIMA forecasting to a majority of ABS time series has seen an improvement in lowering revisions to seasonally adjusted and trend levels and movements, so we will factor this into our experimental studies.

For each time point in the simulation span, constructed so that enough prior and subsequent data points are available to generate a stable estimate, several estimates are calculated at different time lags, reflecting the different levels of data available for calculation of the concurrent estimate. The mean absolute percentage revisions to the seasonally adjusted levels and movements are given, as a function of lag k, respectively by

$$\bar{L}_k = \frac{1}{n_k} \sum_{t=\alpha}^{\omega} \left| L_{t|t+k} \right|$$

and
$$\bar{M}_k = \frac{1}{n_k} \sum_{t=\alpha}^{\omega} \left| M_{t|t+k} \right|,$$

where
$$L_{t|t+k} = 100\left(\widehat{SA}_{t|t+k} - SA_{t|t+k} \right)$$

and
$$M_{t|t+k} = 100\left(\frac{\widehat{SA}_{t|t+k} - \widehat{SA}_{t-1|t+k}}{\widehat{SA}_{t-1|t+k}} - \frac{\widehat{SA}_{t|t+K} - \widehat{SA}_{t-1|t+K}}{\widehat{SA}_{t-1|t+K}} \right),$$

$t = \alpha$ and $t = \omega$ indicate the start and end points of the simulation sub-span respectively, and n_k is the number of observations available in this span at lag k. $\widehat{SA}_{t|t+k}$ denotes the estimate of the seasonally adjusted value at lag k and $\widehat{SA}_{t|t+K}$ is the corresponding stable estimate derived using additional K data points. Mean absolute percentage revisions to level and movement estimates are similarly derived for the trend.

Generally, revisions at early lags should give a good indication of the quality of seasonal adjustment, as it measures the deviation between the initial estimate at time t and its stable estimate given additional data. We shall focus on using revisions at lags 0 and 1 to assess the quality of the seasonal adjustment.

4.2 Residual seasonality

The stable residual seasonality test is one of the most fundamental measures used in assessing the performance of a seasonal adjustment. The presence of seasonality in a series that has supposedly been seasonally adjusted indicates to us a poor seasonal adjustment. Residual seasonality can arise due to various reasons, one of them being the misspecification of a decomposition model. In particular, we are interested in whether seasonal adjustment under a particular Box–Cox transform would produce residual seasonality, indicating its use to be inappropriate.

We define the irregulars as the difference between the trend and the seasonally adjusted series. Here, the trend series is obtained by passing the naïve seasonally adjusted estimates on the original scale through the 13-term Henderson trend filter (with I/C ratio of 1). In testing for stable residual seasonality, an ANOVA is performed on these irregulars. The F-test has the null hypothesis that the factor level (monthly) means are all equal and evidence against this is an indication of residual seasonality.

An issue to consider in regards to the use of this test is the series span on which it is be conducted on. In the extreme case where seasonal spikes and dips are symmetric in magnitude and timing on each end of the seasonally adjusted series, the residual seasonality for that particular period will average out to zero over the entire span, so its presence will be unaccounted for in the F-test. An analysis span of three years at the current end is somewhat too short to detect the presence of residual seasonality. For the purposes of this study, we will restrict the analysis of stable residual seasonality to the last five years.

In addition, the misspecification of decomposition models could also result in *moving* residual seasonality, rather than stable residual seasonality. An F-test for moving residual seasonality is available from the X11 package (Higginson, 1975), though there are limitations to its use in that there are particular cases where seasonality has not been identified. It is highly desired that a conceptually sound test be developed to detect the presence of moving residual seasonality. Here, we acknowledge the HEGY test (Hylleberg *et al.,* 1990) as a possible candidate, in that it individually tests for units roots at different seasonal frequencies. Its appropriate use as a quality measure however, remains a subject of future research, and if proven viable, could potentially benefit our assessment of seasonal adjustment under Box–Cox transforms.

4.3 Relative contribution of volatility to growth (RCVG)

The RCVG measures how much movement of a seasonally adjusted series can be attributed to the irregular component. The larger the RCVG value, the more likely it is that irregular influences are masking the underlying direction of the seasonally adjusted series. The median percentage RCVG value calculated using the entire series span is given by

$$\widetilde{\text{RCVG}}(\%) = median\left(\frac{|\%\Delta I|}{|\%\Delta I| + |\%\Delta T|}\right),$$

where ΔI and ΔT are the month-to-month irregular and trend movements respectively and the irregular component is defined as the difference between the seasonally adjusted and trend estimates.

4.4 Average absolute percentage change (AAPC)

The average absolute percentage change of a seasonally adjusted series provides a measure of its volatility. As opposed to the RCVG measure, the AAPC considers the magnitude of the month-to-month movements, and is given by

$$\text{AAPC} = \frac{1}{N-1}\sum_{t=2}^{N}\left|\frac{SA_t - SA_{t-1}}{SA_{t-1}}\right|.$$

4.5 M7

This summary measure was described in Section 3.4. As mentioned above, we will use the $M7$ criterion only for the purposes of comparing the other estimators with respect to $\hat{\lambda}_{M7}$.

5. ASSESSMENT OF METHODS

In this study, we illustrate the performance of Box–Cox transformation on seasonal adjustment when λ is derived using the method of maximum likelihood, Guerrero's variance stabilisation method, seasonal dummy-variable regression and $M7$ optimisation. The results presented in this paper were calculated based on synthetically simulated as well as real official time series. Knowing the 'true' seasonally adjusted series and λ allows us to make an accurate quality assessment of seasonal adjustment under Box–Cox transformation. An assessment as such is not possible in the context of real data, though we are still interested in comparing the quality measures of the X11-style seasonal adjustment under Box–Cox transformation to those of our standard additive and multiplicative models.

Here we address a number of questions:

- How do the various methods of estimating λ, as outlined in Section 3, perform against one another in terms of quality measures?
- Using quality measures, how do the various methods of estimating λ compare under different levels of volatility?
- How do the seasonally adjusted estimates (on the original scale), derived using $\hat{\lambda}_{MLE}$, $\hat{\lambda}_G$, $\hat{\lambda}_D$ and $\hat{\lambda}_{M7}$, compare with the true seasonally adjusted series (applicable only to simulated data)?

The following sections describe the simulated and real data used, as well as their corresponding settings used in our study. We then present an analysis of the results, addressing the questions posed above.

5.1 Data

5.1.1 Simulated data

In this simulation study, we adopt a procedure in which known trend, seasonal, irregular components have been simulated and the true value of λ used to recover this additive series is also known. The construction of this simulation study can be summarised into two stages. Firstly, we generate $m \times n$ such series, where m is the number of options available for incorporating volatility, σ^2, into the simulation, and n is the number of known λ investigated in this study. As mentioned previously, we will restrict our attention to $0 \le \lambda \le 1$ for practical reasons, that is,

$$\lambda \in \{ 0, 0.2, 0.4, 0.6, 0.8, 1.0 \}; n = 6 .$$

Within `R`, simulated time series were generated by first simulating a seasonal autoregressive time series using `garsim()` from the `gsarima` package. This series was decomposed with an additive option using the `decompose()` moving average decomposition function. The irregular component was then replaced with error terms sampled from independent $N(0,\sigma^2)$ distributions, where the volatility parameter, σ^2, is allowed to vary. We note also that the use of this `decompose()` function automatically satisfies the seasonal balance constraint for additive models.

After constructing the individual components, the original series is then obtained by summing them up according to equation (1). The true seasonally adjusted series on the other hand, is given by the addition of the extracted trend and newly obtained irregular values.

The second stage of this simulation study involves applying reverse-transformation to the original series using the specified values of λ. The same is done for the true seasonally adjusted series. Up until now, all parameters and components have been controlled for. The latter part of the simulation then consists of examining how well the derived estimators of λ lead us back to the true seasonal adjustment figures.

For the seasonal adjustment process, a Box–Cox transform is applied (with λ chosen using each of the methods above) and the seasonally adjusted estimates obtained using X12–ARIMA. This seasonally adjusted series are then returned to the original scale with the naïve inverse Box–Cox transformation using the same $\hat{\lambda}$. Furthermore, as with standard ABS trend methodology, the trend series is subsequently derived using Henderson filters. For more details, see ABS (2003).

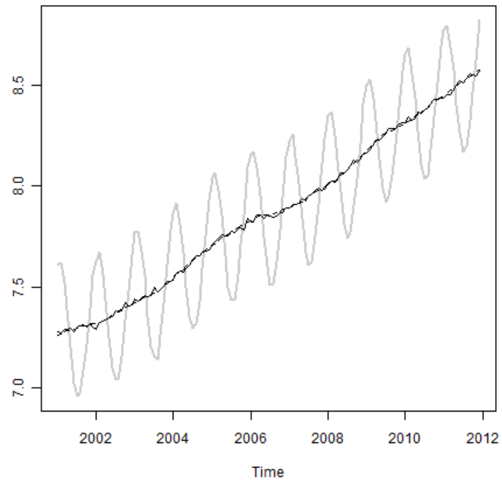
The actual simulation study involved simulating 24 years of monthly time series, with 30 series generated for each of the four volatility settings:

$$\sigma^2 \in \{\ 0.01\,, 0.05\,, 0.10\,, 0.50\ \}\,; m = 4\,.$$

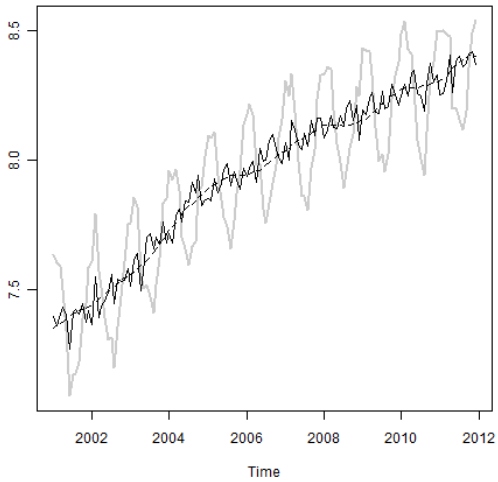
Figure 5.1 illustrates four simulated series, each with a different σ^2.

5.1 Selected plots of simulated series for each volatility setting
(Original series – grey,
Seasonally adjusted series – black,
Trend series – black, dashed)

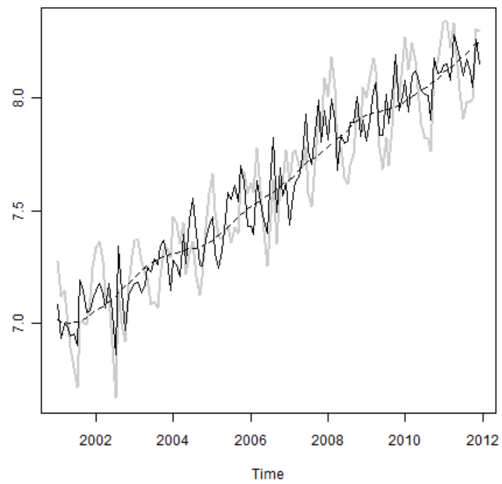
(a) $\sigma^2 = 0.01$



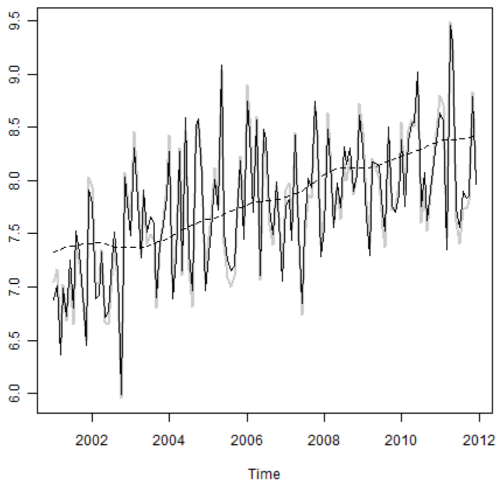
(b) $\sigma^2 = 0.05$



(c) $\sigma^2 = 0.1$



(d) $\sigma^2 = 0.5$



5.1.2 Real data

As mentioned in Section 1, we will investigate a number of Retail Trade and Short Term Overseas Arrivals and Departures (OAD) series as potential candidates for Box–Cox transformation. In particular, they are:

1. Retail Trade – Australia Total;
2. Retail Trade – Western Australia Total;
3. Retail Trade – Australia Newspaper and Book Retailing;
4. OAD – Departures to France;
5. OAD – Departures to Nepal; and
6. OAD – Arrivals from Indonesia.

All six of these series are currently analysed using a multiplicative model. For more details regarding these series, refer to ABS (2011a) and ABS (2011b).

In our literature review, we outlined an issue concerning outlier detection when Box–Cox transformation is incorporated into the seasonal adjustment process. It may well be the case that outliers and/or calendar effects (such as trading day, Easter, Ramadan) on the original scale end up undetected or magnified on the transformed scale. Instead of attempting to estimate these effects on the transformed scale, we carry out Box–Cox transformation on the 'cleaned' original data, that is, data that has been corrected for outliers and/or calendar effects. As with the simulated data, the resultant transformed series is then fed into the seasonal adjustment process to extract the estimated seasonally adjusted series and back-transformed to the original scale. The trend series is also derived for these real datasets.

5.2 Results

5.2.1 Simulated data

The mean square error (MSE) of both the seasonally adjusted and trend estimates, residual seasonality tests and revision quality measures taken over these simulations for the four methods of estimating λ are shown in the table below. In addition, the "Best of 0, 1" method and the true value of λ used in the simulation also have their performance indicated, for the purposes of comparison. "Best of 0, 1" refers to selecting λ either zero or one for each series depending on what gives the best result, for each series and for each quality measure.

For each simulated series the performance of each λ is given as a percentage of the best (lowest) value achieved over all tested λ values. For example, in table 5.2(a), $\hat{\lambda}_{M7}$ gives a MSE ratio of 103.95 for a true λ value of 0.6. That is, for those simulations with a true λ value of 0.6, $\hat{\lambda}_{M7}$ achieved an MSE on average 3.95% higher than the best MSE achieved for those series.

The tables below show that for most measures, the estimator $\hat{\lambda}_{M7}$ achieves good results. Perhaps surprisingly the quality of seasonal adjustment when using the maximum likelihood method is similar to the quality of seasonal adjustment when the true λ is used.

Considering only the four estimators, a simple ranking of their performances suggest $\hat{\lambda}_{M7}$ as the best estimator, followed by $\hat{\lambda}_G$, $\hat{\lambda}_{MLE}$ and lastly, $\hat{\lambda}_D$ (table 5.12). For more details regarding the simple ranking method, refer to Appendix B. Technically speaking, a statistical test such as the Wilcoxon signed-rank test should be performed in order to make more concrete comparisons of the estimators, but that would require more simulated series for each fixed volatility setting, which we currently do not have. Future investigations should factor this into account.

The most obvious cases of poor seasonal adjustment occur when an additive or multiplicative transform is inappropriately applied. The results emphasise that the additive or multiplicative transform are both not adequate for series which are generated using transforms other than λ being zero or one. Such inappropriate choices give rise to high MSE as indicated in table 5.2(a). In addition, tables 5.3 – 5.11 show that the performance is worse when an additive decomposition is applied to a multiplicatively simulated series, more so than when a multiplicative decomposition is applied to an additively simulated series.

5.2(a) MSE performance (seasonally adjusted) relative to best results for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	**120.41**	106.86	105.63	104.14	103.58	**102.86**
Guerrero	125.67	**106.78**	105.24	104.02	103.61	103.59
Dummy	184.62	109.60	106.26	107.66	111.73	117.27
M7	123.44	106.93	**104.92**	**103.95**	**103.40**	103.15
Best of 0, 1	*105.10*	*129.83*	*142.89*	*127.07*	*107.82*	*101.25*
True	*121.68*	*106.25*	*104.39*	*103.20*	*102.79*	*102.75*

5.2(b) MSE performance (trend) relative to best results for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	102.68	101.91	101.49	101.27	**101.09**	101.06
Guerrero	102.88	101.92	101.52	101.28	101.10	100.94
Dummy	149.70	103.57	102.71	103.04	105.26	109.38
M7	**102.46**	**101.87**	**101.47**	**101.25**	101.14	**100.91**
Best of 0, 1	*102.23*	*111.09*	*117.68*	*111.27*	*102.37*	*100.61*
True	*102.67*	*101.94*	*101.59*	*101.37*	*101.19*	*101.04*

5.3 Percentage of series with $p<0.05$ for residual seasonality F-test

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	0	0	0	0	0	0
Guerrero	0	0	0	0	0	0
Dummy	0	0	0	0	0	0
M7	0	0	0	0	0	0
0	*0*	*0*	*0*	*0*	*0*	*0*
1	*10.8*	*0*	*0*	*0*	*0*	*0*
True	*0*	*0*	*0*	*0*	*0*	*0*

5.4 Revisions to seasonally adjusted level (from lag 0), relative to lowest achieved for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
ML	103.96	103.16	103.40	103.31	102.81	102.34
Guerrero	103.71	**102.74**	102.91	**102.85**	**102.53**	102.33
Dummy	127.22	106.58	105.36	106.91	109.70	114.13
M7	**103.61**	102.55	**102.69**	102.90	102.62	**102.23**
0	*103.53*	*123.77*	*130.92*	*134.13*	*135.74*	*136.29*
1	*512.07*	*222.76*	*153.60*	*124.34*	*109.07*	*102.42*
True	*103.53*	*102.76*	*102.98*	*102.96*	*102.65*	*102.42*

5.5 Revisions to seasonally adjusted movement (from lag 0) relative to lowest achieved for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	103.65	103.75	103.95	103.64	103.32	102.79
Guerrero	**102.82**	103.11	103.35	**103.15**	**102.85**	102.68
Dummy	109.07	104.62	104.52	104.72	105.16	106.72
M7	103.00	**103.03**	**103.11**	103.21	102.86	**102.66**
0	*103.15*	*110.94*	*113.06*	*114.00*	*114.27*	*114.45*
1	*332.41*	*148.69*	*123.23*	*112.82*	*106.84*	*102.96*
True	*103.15*	*103.43*	*103.59*	*103.38*	*103.14*	*102.96*

5.6 Revisions to trend level (from lag 0) relative to lowest achieved for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	**102.58**	**102.27**	**102.17**	**101.90**	**101.58**	**101.36**
Guerrero	102.78	102.35	102.20	101.92	101.61	101.40
Dummy	114.38	103.73	103.28	103.83	104.61	106.47
M7	102.72	102.39	102.18	101.98	101.64	101.47
0	*102.58*	*111.41*	*114.64*	*116.22*	*117.00*	*117.49*
1	*322.13*	*161.89*	*124.09*	*110.21*	*103.65*	*101.38*
True	*102.58*	*102.28*	*102.18*	*101.90*	*101.59*	*101.38*

5.7 Revisions to trend movement (from lag 0) relative to lowest achieved for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	**102.33**	**101.75**	101.47	101.29	**101.07**	100.90
Guerrero	102.56	101.80	**101.45**	**101.29**	101.08	100.86
Dummy	106.97	102.12	101.79	101.64	101.87	101.97
M7	102.44	101.82	101.48	101.33	101.10	**100.84**
0	*102.45*	*104.39*	*105.03*	*105.45*	*105.61*	*105.63*
1	*204.12*	*124.14*	*107.82*	*103.28*	*101.48*	*100.91*
True	*102.45*	*101.82*	*101.53*	*101.36*	*101.13*	*100.91*

5.8 Revisions to seasonally adjusted level (from lag 1) relative to lowest achieved for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	104.00	103.55	103.53	103.37	102.96	**102.28**
Guerrero	103.67	103.08	103.12	103.08	102.87	102.65
Dummy	128.62	108.13	106.07	108.37	111.32	117.13
M7	**103.52**	**102.91**	**102.84**	**103.05**	**102.71**	102.42
0	*103.46*	*128.25*	*135.29*	*138.77*	*140.70*	*141.50*
1	*528.40*	*235.29*	*159.57*	*128.11*	*110.93*	*102.47*
True	*103.46*	*103.07*	*103.05*	*102.95*	*102.73*	*102.47*

5.9 Revisions to seasonally adjusted movement (from lag 1) relative to lowest achieved for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	103.54	103.73	103.88	103.58	103.31	**102.64**
Guerrero	**102.76**	103.22	103.42	**103.20**	**102.95**	102.84
Dummy	110.00	104.57	104.14	104.65	105.57	107.08
M7	102.90	**103.17**	**103.36**	103.24	102.99	102.85
0	*102.96*	*110.97*	*113.03*	*114.23*	*114.69*	*114.95*
1	*339.48*	*152.66*	*124.25*	*112.70*	*106.56*	*102.82*
True	*102.96*	*103.31*	*103.44*	*103.20*	*103.04*	*102.82*

5.10 Revisions to trend level (from lag 1) relative to lowest achieved for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	104.06	103.19	103.16	102.90	**102.49**	**102.03**
Guerrero	104.16	103.12	103.05	**102.78**	102.50	102.23
Dummy	129.31	106.84	105.59	107.52	110.05	115.07
M7	**103.95**	**103.12**	**103.02**	102.90	102.50	102.32
0	*103.85*	*125.77*	*132.70*	*136.10*	*137.88*	*138.81*
1	*513.64*	*225.34*	*153.60*	*124.83*	*108.87*	*102.12*
True	*103.85*	*103.01*	*102.99*	*102.72*	*102.40*	*102.12*

5.11 Revisions to trend movement (from lag 1) relative to lowest achieved for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	**102.56**	**102.01**	101.96	101.74	**101.44**	101.26
Guerrero	102.74	102.05	**101.92**	**101.70**	101.46	101.27
Dummy	113.48	103.19	102.77	102.98	103.63	104.82
M7	102.71	102.05	101.93	101.76	101.46	**101.24**
0	*102.61*	*109.00*	*111.53*	*112.86*	*113.54*	*113.94*
1	*303.32*	*155.66*	*120.21*	*108.02*	*102.89*	*101.27*
True	*102.61*	*102.01*	*101.94*	*101.73*	*101.46*	*101.27*

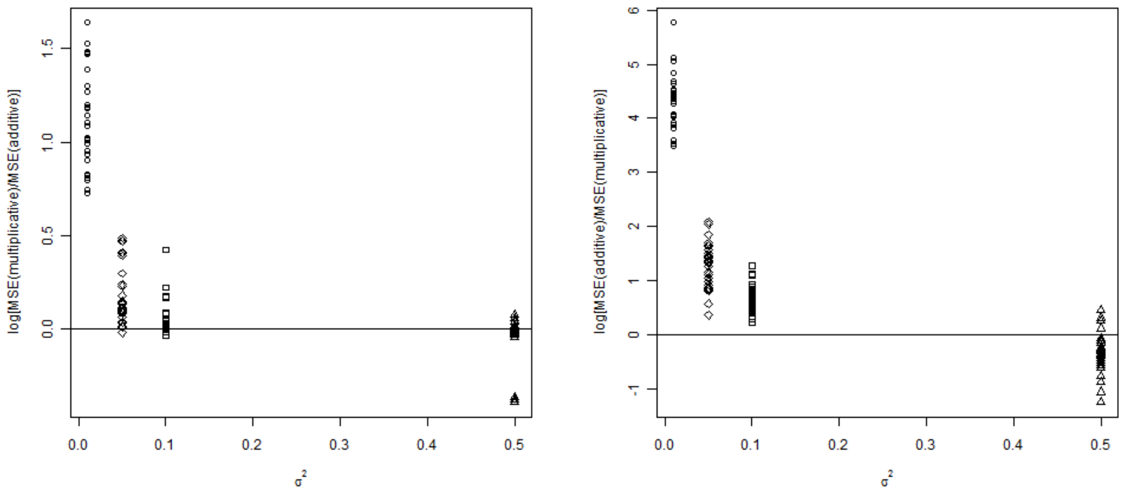
5.12 Ranking of estimator performance (1-Best, 4-Worst)

	Estimator			
Performance / Quality Measure	*ML*	*Guerrero*	*M7*	*Dummy*
MSE of SA	2	3	1	4
MSE of T	2	3	1	4
Revisions to SA level lag 0	3	1 *	1 *	4
Revisions to SA movement lag 0	3	1 *	1 *	4
Revisions to T level lag 0	1	2	3	4
Revisions to T movement lag 0	1	2	3	4
Revisions to SA level lag 1	3	2	1	4
Revisions to SA movement lag 1	3	1	2	4
Revisions to T level lag 1	1 *	1 *	1 *	4
Revisions to T movement lag 1	1	2 *	2 *	4
Final rank	3	2	1	4

In terms of MSE for multiplicative simulation the best performance sometimes came from using λ of 1 rather than 0. This can be seen by the MSE for the true λ being higher than the "Best of 0,1" method. Further investigation showed that this occurred for multiplicatively simulated series with the highest volatility setting. This relationship between best fitting decomposition model and level of volatility can be seen in the plots below.

Figure 5.13(L) displays the log ratios of the MSE under a multiplicative model to the MSE under an additive model, when the true transform is additive, for each volatility option. Conversely, figure 5.13(R) displays the log ratios of the MSE under an additive model to the MSE under a multiplicative model, when the true transform is multiplicative. One would expect all points to be above the horizontal line, meaning that choosing whichever value of λ was used to simulate the series (here only 0 or 1) results in reduced MSE. However, for the very volatile series this is not true and in fact a smaller MSE is achieved by choosing an additive model even when a multiplicative model generated the series. Whilst this may appear to support using an additive model for all highly volatile series, the results in table 5.3 show that using an additive decomposition model for a series that is truly log-additive can introduce residual seasonality.

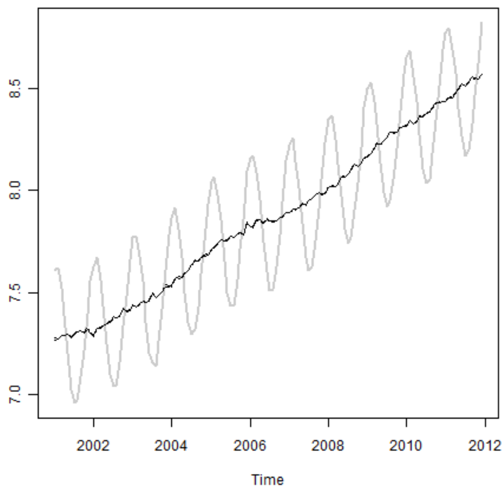
5.13 Log ratios of MSEs (of seasonally adjusted) versus volatility when true model is additive (L) and when the true model is multiplicative (R)



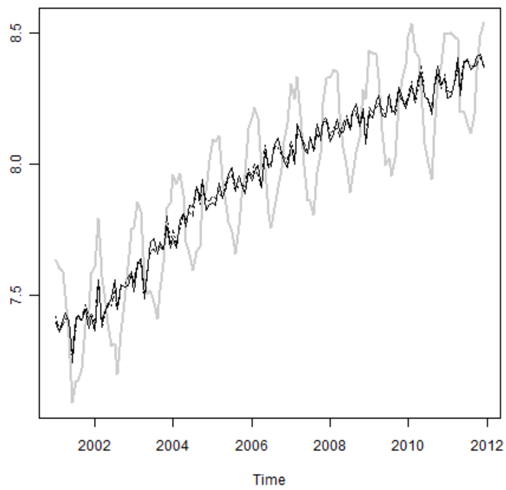
Figures 5.14–5.15 display selected plots of either additively or multiplicatively simulated series and their corresponding true and estimated seasonally adjusted series. On average, there are larger discrepancies between the true and estimated seasonally adjusted estimates for a highly volatile multiplicatively simulated series.

5.14 Selected plots of additively simulated series for each volatility setting
(Original series – grey,
True seasonally adjusted – black,
Estimated seasonally adjusted – black, dashed)

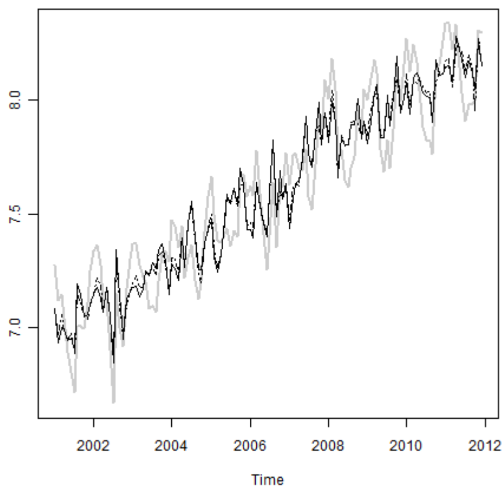
(a) $\sigma^2 = 0.01$



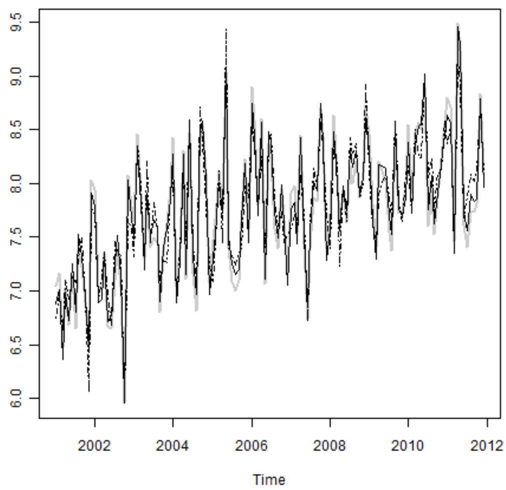
(b) $\sigma^2 = 0.05$



(c) $\sigma^2 = 0.1$

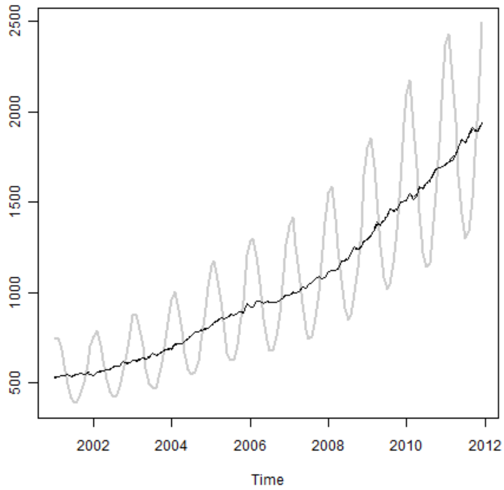


(d) $\sigma^2 = 0.5$

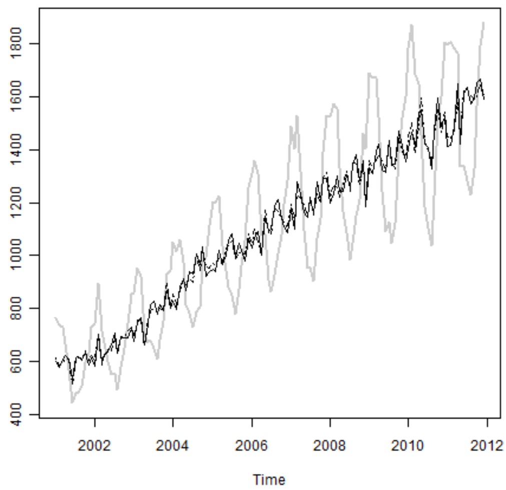


5.15 Selected plots of multiplicatively simulated series for each volatility setting
(Original series – grey,
True seasonally adjusted – black,
Estimated seasonally adjusted – black, dashed)

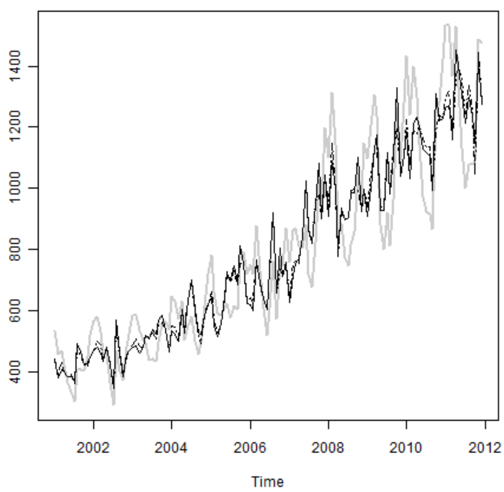
(a) $\sigma^2 = 0.01$



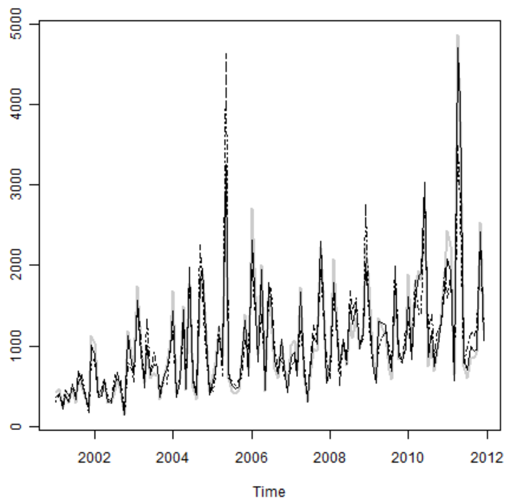
(b) $\sigma^2 = 0.05$



(c) $\sigma^2 = 0.1$



(d) $\sigma^2 = 0.5$



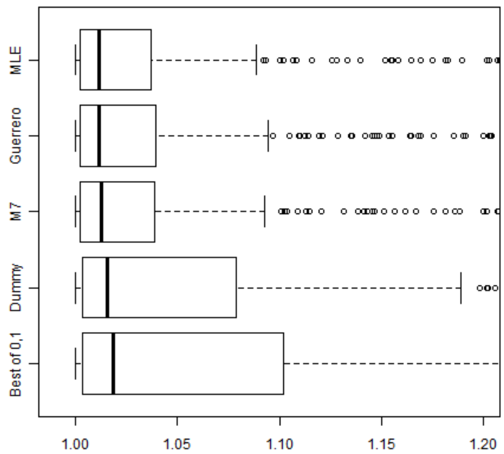
An alternative assessment that is more robust against the impact of the highly volatile series is to produce boxplots summarising the ratios of each method to the best method for each quality measure where the best transform is found by a grid search of the values $\{0.0, 0.1, \ldots, 1.0\} \cup \left\{\hat{\lambda}_{MLE}, \hat{\lambda}_G, \hat{\lambda}_D, \hat{\lambda}_{M7}\right\}$.

The boxplots below have been graphically truncated to enable the comparison of medians between each method. Note that in each quality measure, the boxplots are heavily right-skewed. This is mainly due to the impact of series with a volatility setting of 0.5.

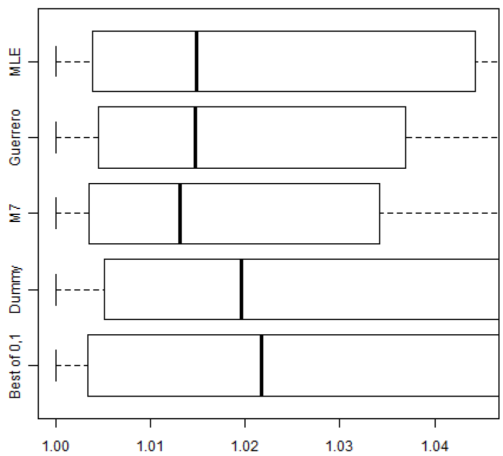
Methods with the median placed closer to 1 are those which are more often closer to the best available. It can be seen in each of the revision boxplots that $\hat{\lambda}_D$ performs rather poorly compared to $\hat{\lambda}_G$, $\hat{\lambda}_{MLE}$ and $\hat{\lambda}_{M7}$, all of which seem to perform similarly against one another, with $\hat{\lambda}_{M7}$ slightly outperforming $\hat{\lambda}_G$ and $\hat{\lambda}_{MLE}$.

5.16 Performance relative to best transform

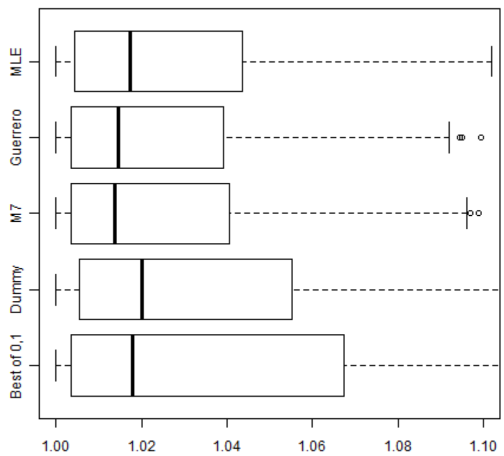
(a) MSE



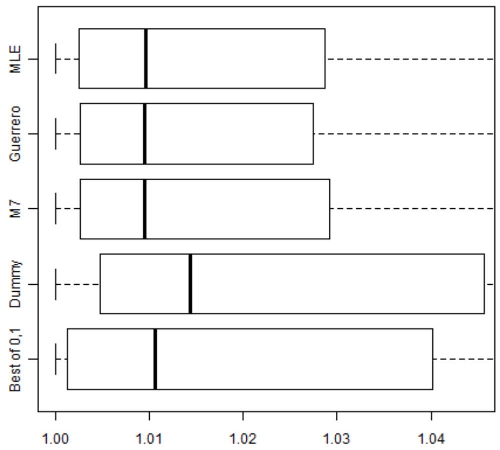
(b) Revisions to SA level estimates (lag 0)



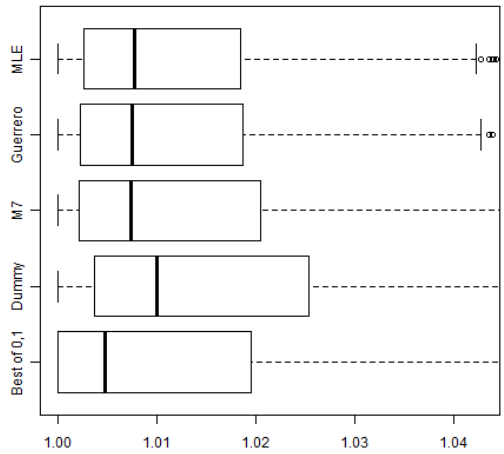
(c) Revisions to SA movement estimates (lag 0)



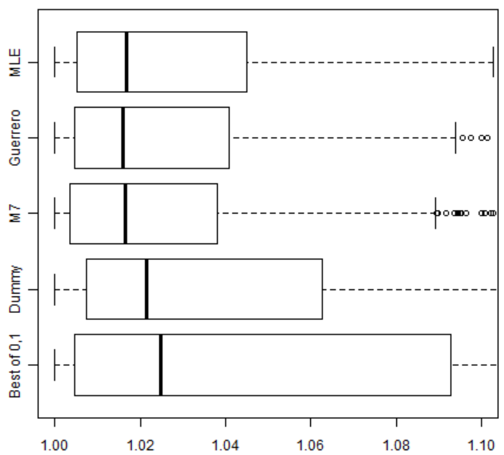
(d) Revisions to T level estimates (lag 0)



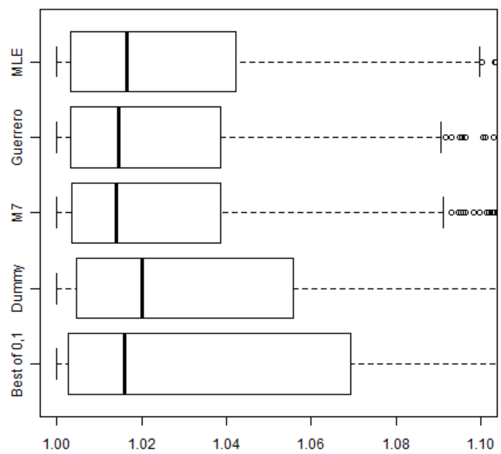
(e) Revisions to T movement estimates (lag 0)



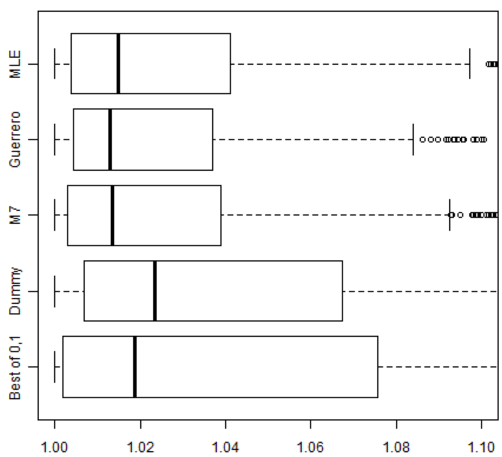
(f) Revisions to SA level estimates (lag 1)



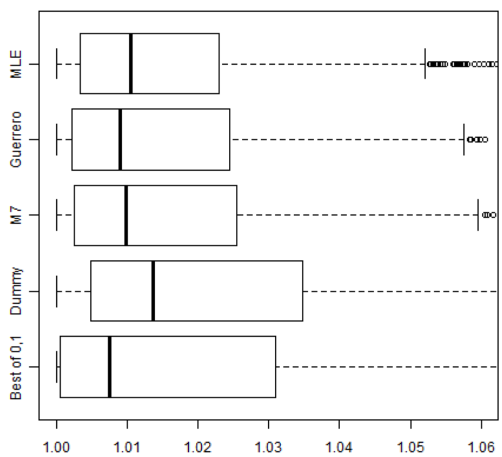
(g) Revisions to SA movement estimates (lag 1)



(h) Revisions to T level estimates (lag 1)



(i) Revisions to T movement estimates (lag 1)



5.2.2 Real data

Results are presented below showing some improvement to the usual seasonal adjustment methods in some cases. All six quality measures as discussed above have been computed and displayed graphically for each series. From left to right, top to bottom, the AAPC, RCVG, $M7$, residual seasonality probabilities and revisions to seasonally adjusted and trend movements and levels are provided as a function of the grid values of λ. In the revision graphs, the solid line denotes the revisions at lag 0 whereas the dotted line corresponds to lag 1. Most cases, as expected, indicate lower revisions for lag 1 than for lag 0.

For each of these series, tables 5.19, 5.23, 5.27, 5.31, 5.35 and 5.40 display the performance of each estimator with respect to the existing multiplicative adjustment. The percentage performance is calculated as

$$\left(1-\frac{QM\left(\lambda=\hat{\lambda}\right)}{QM\left(\lambda=0\right)}\right)\times 100\%$$

where QM is the quality measure of interest and $\hat{\lambda}\in\left\{\hat{\lambda}_{MLE},\hat{\lambda}_{G},\hat{\lambda}_{M7},\hat{\lambda}_{D}\right\}$.

A more detailed summary of the quality measures for each grid value of λ as well as these four estimators can be found in Appendix C.

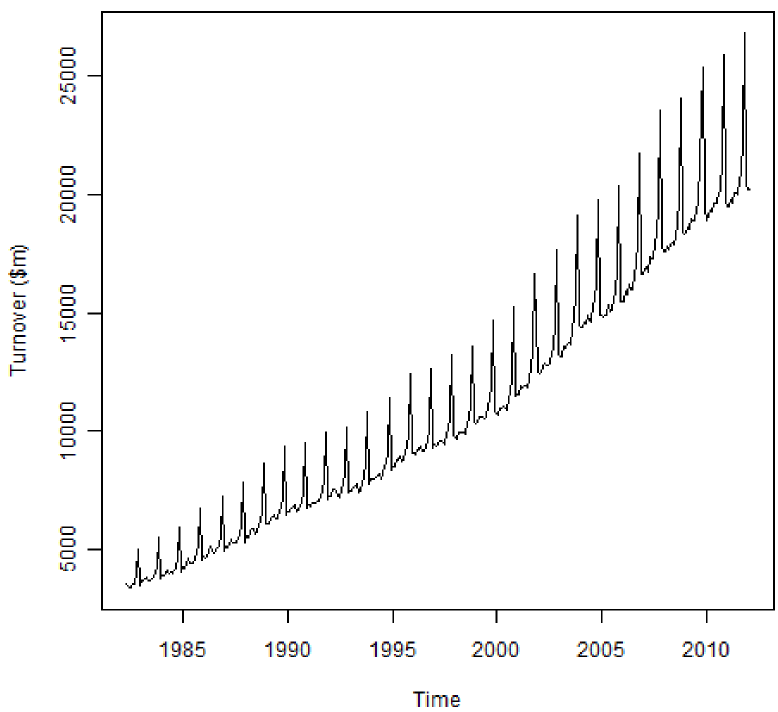
Retail Trade – Australia Total

There is some evidence that improvements can be made to the seasonal adjustment of the headline Retail series with $0 \le \hat{\lambda} \le 0.4$ (Appendix C.1(b)). This is particularly indicated in the $M7$ plot, for which all four estimators show a 31.8–42.4% improvement in the $M7$ indicator with respect to the multiplicative adjustment (figure 5.18). However, there are no substantial gains to be made in regards to the revision performance of these estimators, with a 0.5–2.9% reduction in revisions to seasonally adjusted and trend level estimates but 3.2–6.1% increase in revisions to the respective movement estimates, for the case of $\hat{\lambda}_G = 0.22$.

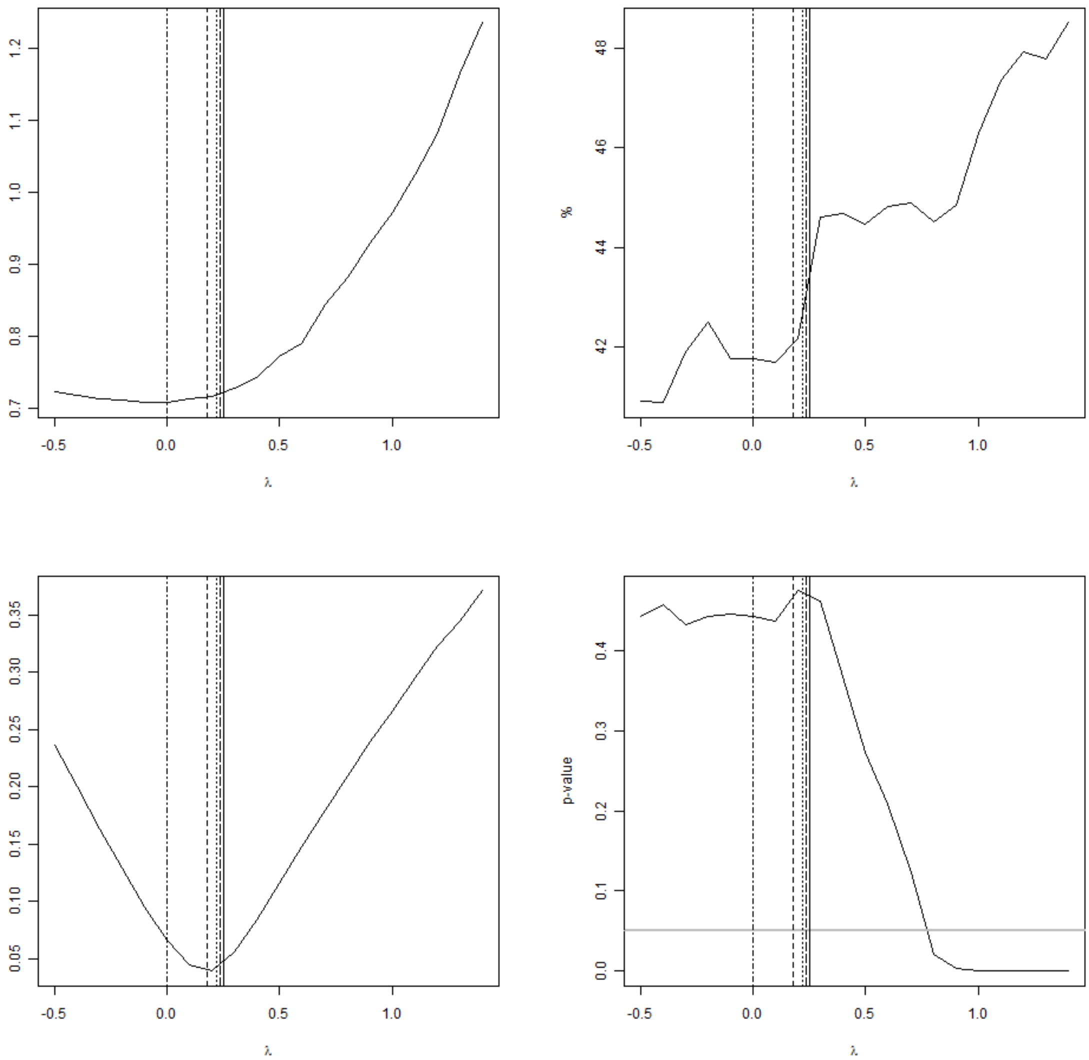
Considering all four proposed estimators, $\hat{\lambda}_{M7} = 0.17$ performed the best across all quality measures, followed by $\hat{\lambda}_G$ and $\hat{\lambda}_{MLE}$, which both yield similar results. The worst performing estimator was $\hat{\lambda}_D$ (see table 5.19).

From the residual seasonality plot (figure 5.18), it can be seen that the significance of residual seasonality increases with higher values of λ. In particular, $\lambda \ge 0.7$ yielded significant p-values at the 5% level.

5.17 Retail Trade – Australia Total



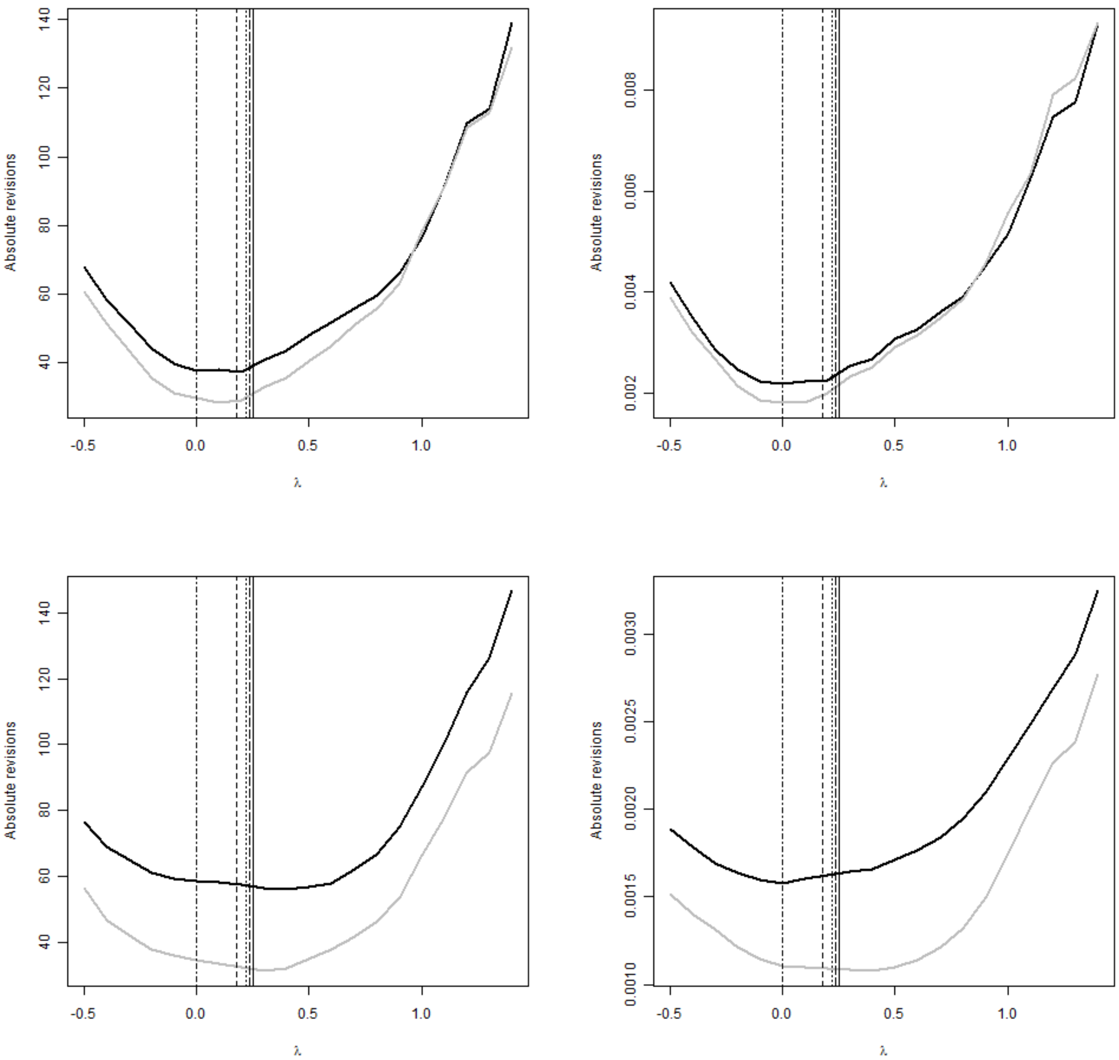
5.18 Retail Trade – Australia Total, Quality Measures against lambda values
(from L–R, T–B, AAPC, RCVG, M7 and residual seasonality p-value: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))



5.19 Retail Trade – Australia Total, Percentage performance of estimators with respect to multiplicative adjustment

	Estimator			
Quality Measure	*MLE (0.23)*	*Guerrero (0.22)*	*Dummy (0.25)*	*M7 (0.17)*
AAPC	-1.4	-1.4	-1.6	-1.0
RCVG	-3.2	-2.6	-4.8	0.5
M7	**34.8**	**37.9**	**31.8**	**42.4**
Revisions to SA level lag 0	-0.6	0.5	-2.5	1.8
Revisions to SA movement lag 0	-7.9	-6.1	-10.2	-0.9
Revisions to T level lag 0	**2.7**	**2.1**	**2.9**	**1.6**
Revisions to T movement lag 0	-3.2	-3.2	-3.6	-2.3

5.20 Retail Trade – Australia Total,
Lag 0 (black) and lag 1 (grey) Revision Measures against lambda values
(from L–R, T–B, seasonally adjusted level, seasonally adjusted movement, trend level and trend movement estimates: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))



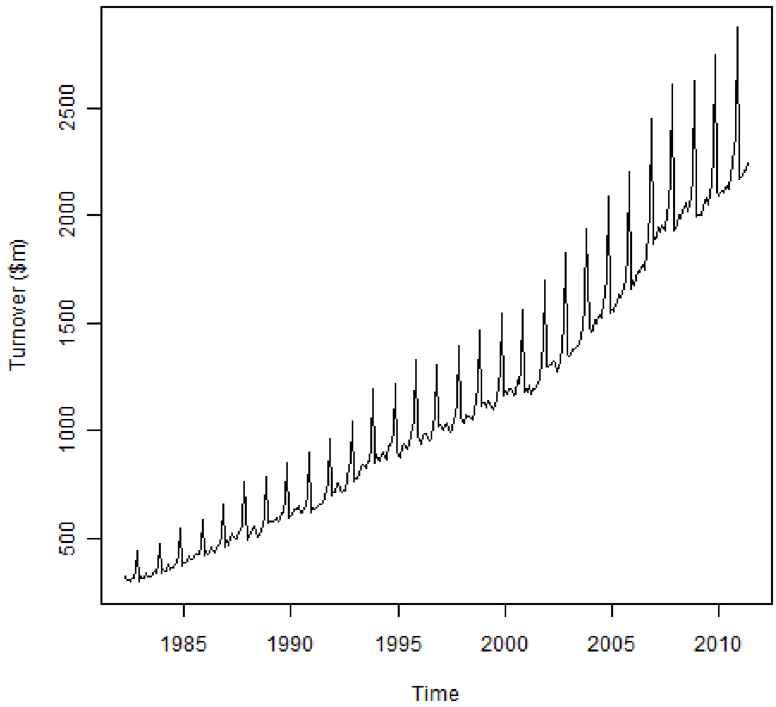
Retail Trade – Western Australia Total

As with the Australia Total, there are some benefits to using Box–Cox transformation for the Western Australia series. In particular, $0.1 \le \hat{\lambda} \le 0.2$ yielded better reductions in the revisions to the seasonally adjusted level and movement estimates. Here, $\hat{\lambda}_{M7} = 0.12$ was shown to outperform the other estimators on the basis of the volatility and seasonally adjusted and trend movement estimates (Appendix C.2(a) and C.2(b)). This is then followed by $\hat{\lambda}_G = 0.18$, $\hat{\lambda}_D = 0.21$ and lastly $\hat{\lambda}_{MLE} = 0.23$.

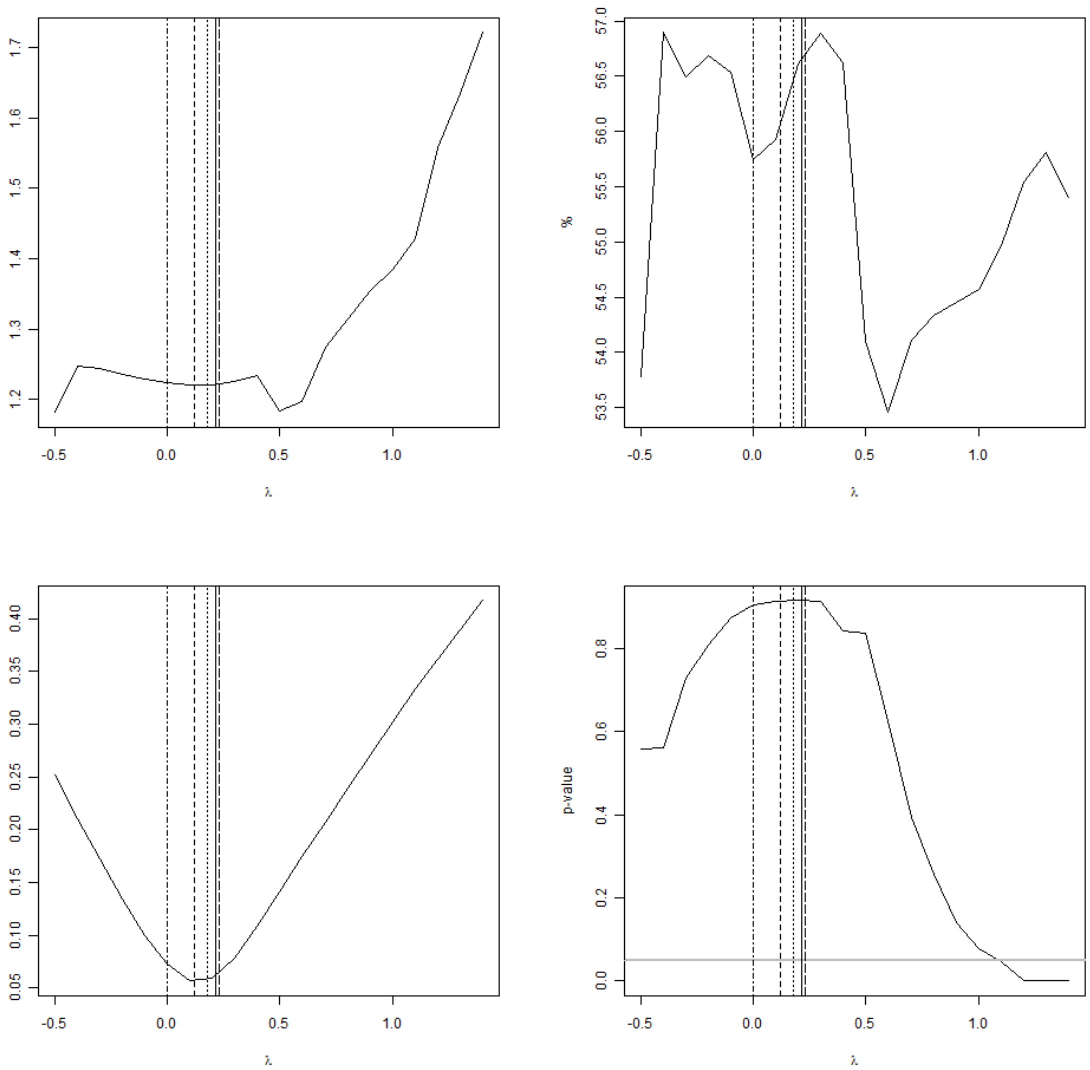
Comparing with the multiplicative adjustment specifically, all four estimators presented 12.3–23.3% gains in $M7$ performance, as well as 3.2–8.8% reduction in the revisions to seasonally adjusted and trend level estimates. However, like the Australia Total series, the estimators reported increases in the revisions to the respective movement estimates, ranging from 0.1–9.2% (see table 5.23).

For this series, Box–Cox transforms of $\lambda \ge 1.1$ resulted in poor quality seasonal adjustments, with residual seasonality present at the 5% significance level (figure 5.22).

5.21 Retail Trade – Western Australia Total



5.22 Retail Trade – Western Australia Total, Quality Measures against lambda values
(from L–R, T–B, AAPC, RCVG, M7 and residual seasonality p-value: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))

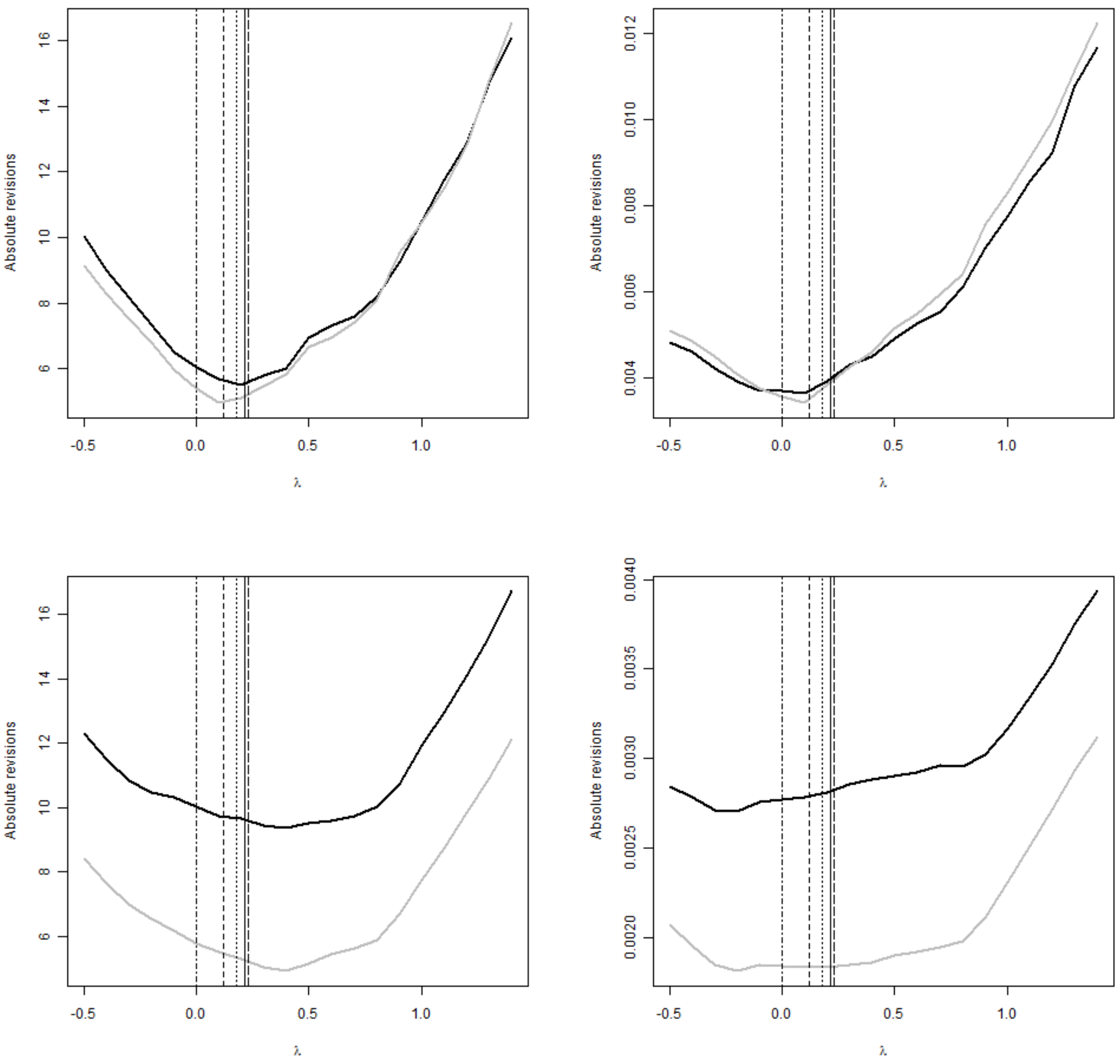


5.23 Retail Trade – Western Australia Total, Percentage performance of estimators with respect to multiplicative adjustment

	Estimator			
Quality Measure	*MLE (0.23)*	*Guerrero (0.18)*	*Dummy (0.21)*	*M7 (0.12)*
AAPC	**0.3**	**0.3**	**0.3**	**0.4**
RCVG	-1.5	-1.4	-1.5	-1.1
M7	**12.3**	**21.9**	**16.4**	**23.3**
Revisions to SA level lag 0	**8.8**	**8.8**	**8.5**	**6.2**
Revisions to SA movement lag 0	-9.2	-4.9	-7.2	-0.1
Revisions to T level lag 0	**4.7**	**3.3**	**4.0**	**3.2**
Revisions to T movement lag 0	-1.5	-1.3	-1.5	-0.3

5.24 Retail Trade – Western Australia Total, Lag 0 (black) and lag 1 (grey) Revision Measures against lambda values

(from L–R, T–B, seasonally adjusted level, seasonally adjusted movement, trend level and trend movement estimates: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))

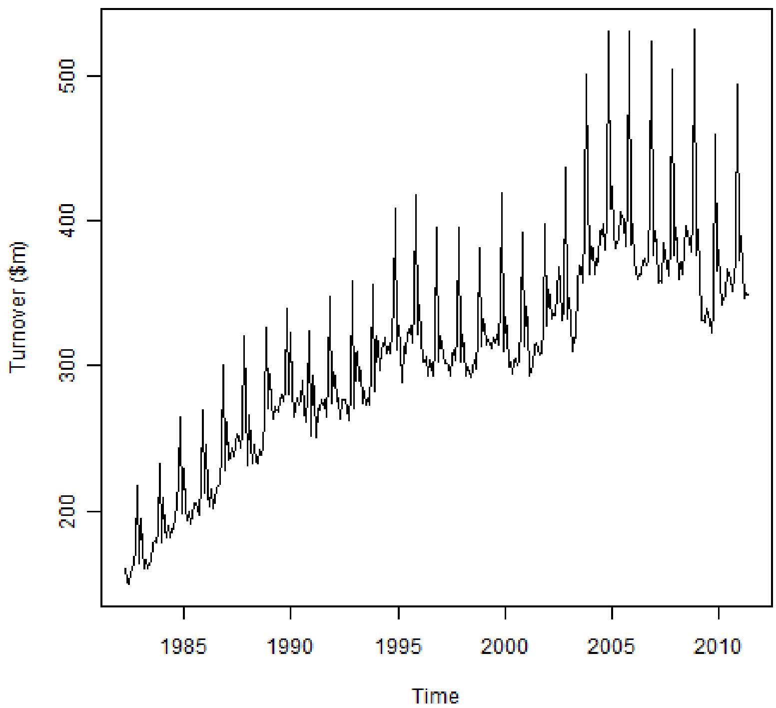


Retail Trade – Newspapers and Book Retailing

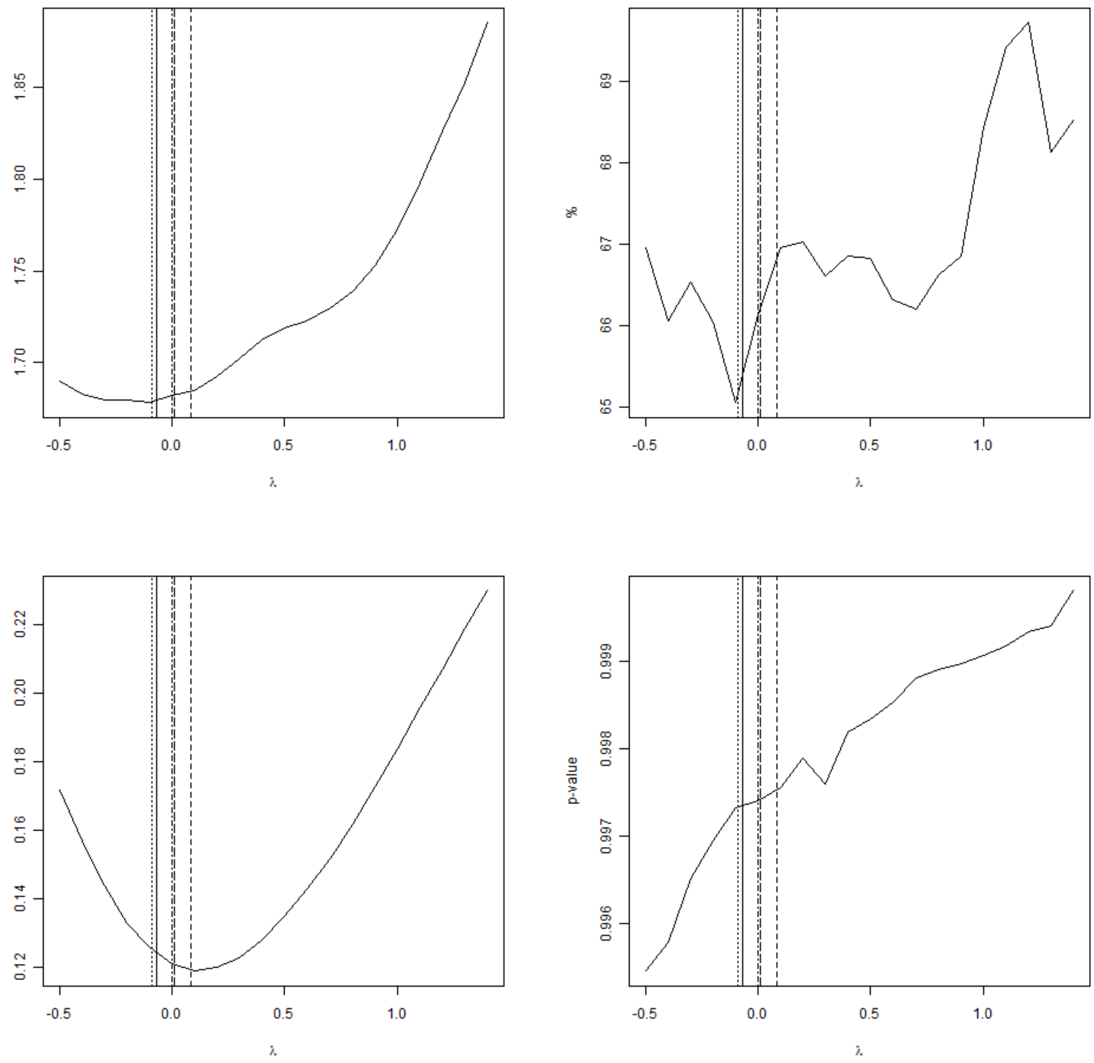
Results for this series indicate that $-0.4 \le \hat{\lambda} \le -0.1$ can lead to better quality seasonal adjustment estimates. In particular, the grid search found $\hat{\lambda} = -0.1$ yielding the best results for AAPC, RCVG and revisions to seasonally adjusted level estimates (Appendix C.3(a) and C.3(b)). Furthermore, the estimators $\hat{\lambda}_G$ and $\hat{\lambda}_D$ both reported values of approximately – 0.1, and was shown to perform better across all quality measures (except $M7$) than the current multiplicative adjustment (table 5.27).

There is no strong evidence for residual seasonality (in the last five years) in the seasonally adjusted estimates under Box–Cox transformation (see figure 5.26). We also visually inspected each seasonally adjusted series in search for odd patterns, possibly arising due to the unconventional choice of λ, but found none that were alarming.

5.25 Retail Trade – Newspapers and Book Retailing



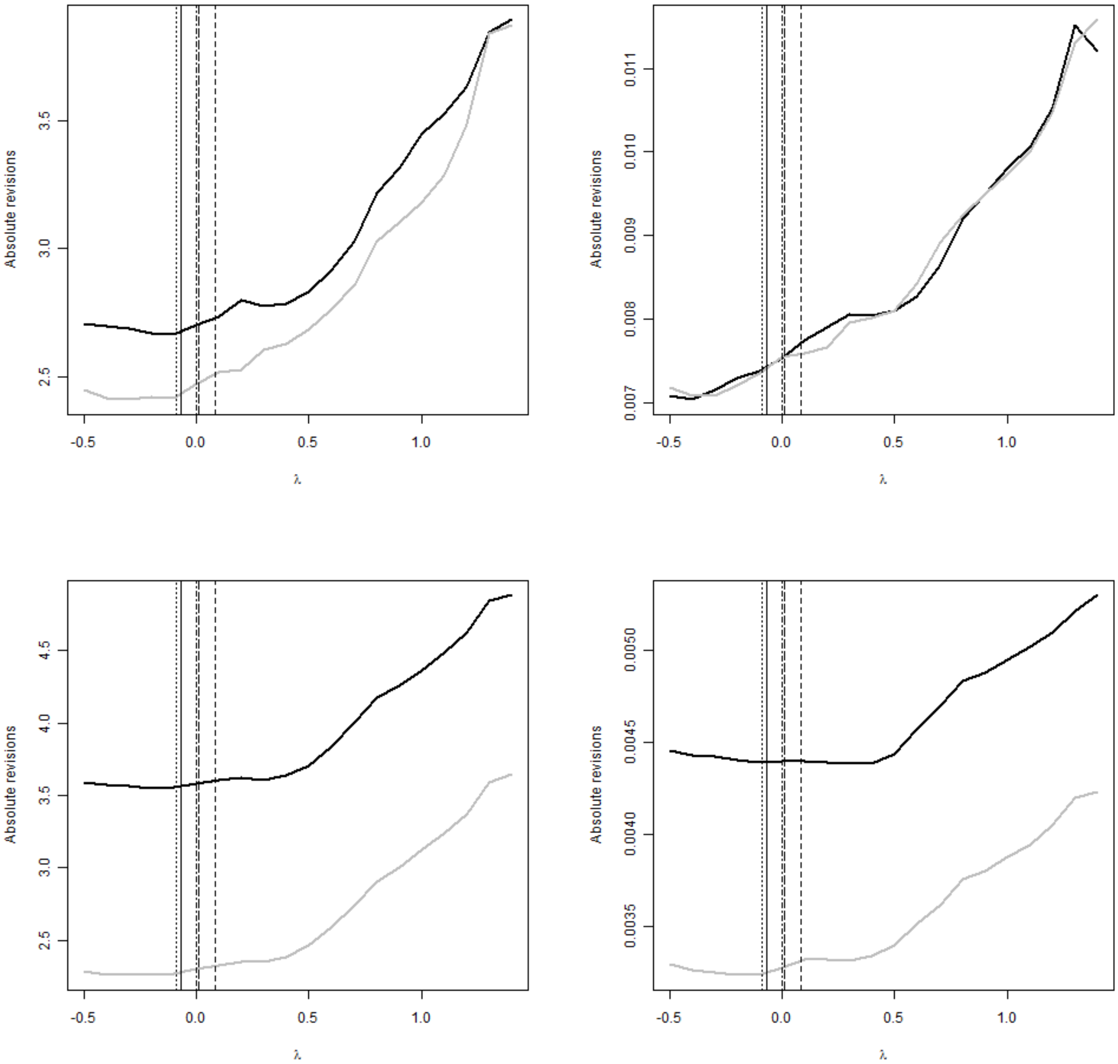
5.26 Retail Trade – Newspapers and Book Retailing, Quality Measures against lambda values
(from L–R, T–B, AAPC, RCVG, M7 and residual seasonality p-value: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))



5.27 Retail Trade – Newspapers and Book Retailing, Percentage performance of estimators with respect to multiplicative adjustment

	Estimator			
Quality Measure	*MLE (0.01)*	*Guerrero (-0.09)*	*Dummy (-0.07)*	*M7 (0.08)*
AAPC	0	**0.2**	**0.2**	-0.1
RCVG	**0.4**	**1.5**	**1.3**	-1.2
M7	0	-3.3	-2.5	**1.7**
Revisions to SA level lag 0	0	**1.3**	**1.4**	-1.2
Revisions to SA movement lag 0	-0.3	**1.8**	**1.9**	-2.5
Revisions to T level lag 0	0	**0.6**	**0.4**	-0.5
Revisions to T movement lag 0	0	**0.2**	**0.2**	-0.2

5.28 Retail Trade – Newspapers and Book Retailing,
Lag 0 (black) and lag 1 (grey) Revision Measures against lambda values
(from L–R, T–B, seasonally adjusted level, seasonally adjusted movement, trend level and trend movement estimates: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))



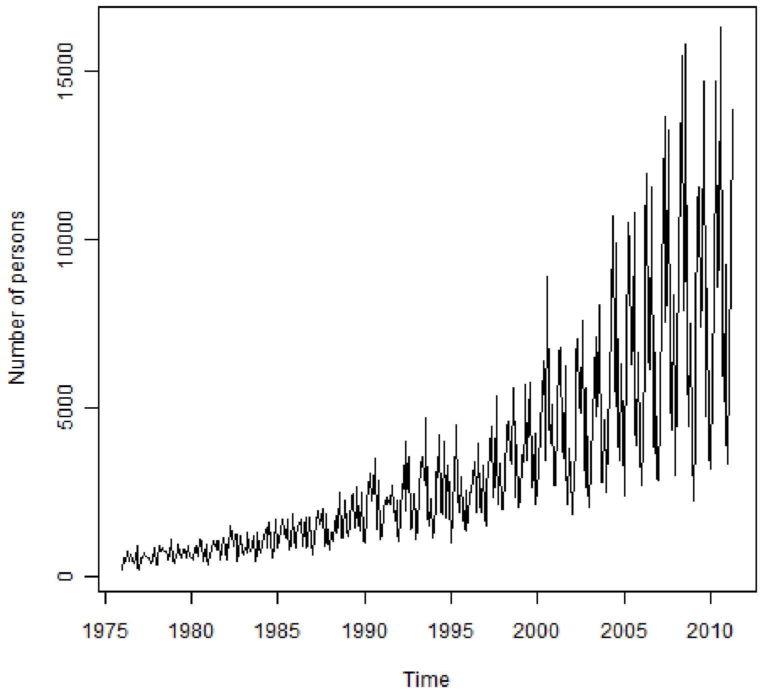
OAD – Departures to France

The seasonality of this series increases vastly with the level of the series. This behaviour makes this series a candidate for transformation with a negative value of λ. In fact, the grid search found $\hat{\lambda} = -0.5$ most optimal in term of improving revision performance (Appendix C.4(b)). Compared to the multiplicative adjustment, this transform achieved 28.3% and 30.2% reductions to revisions of the level seasonally adjusted and trend estimates respectively. On the other hand, the multiplicative transform yielded volatility measures which were marginally better than of $\hat{\lambda} = -0.5$, as well as achieving a slightly lower $M7$ value (Appendix C.4(a)).

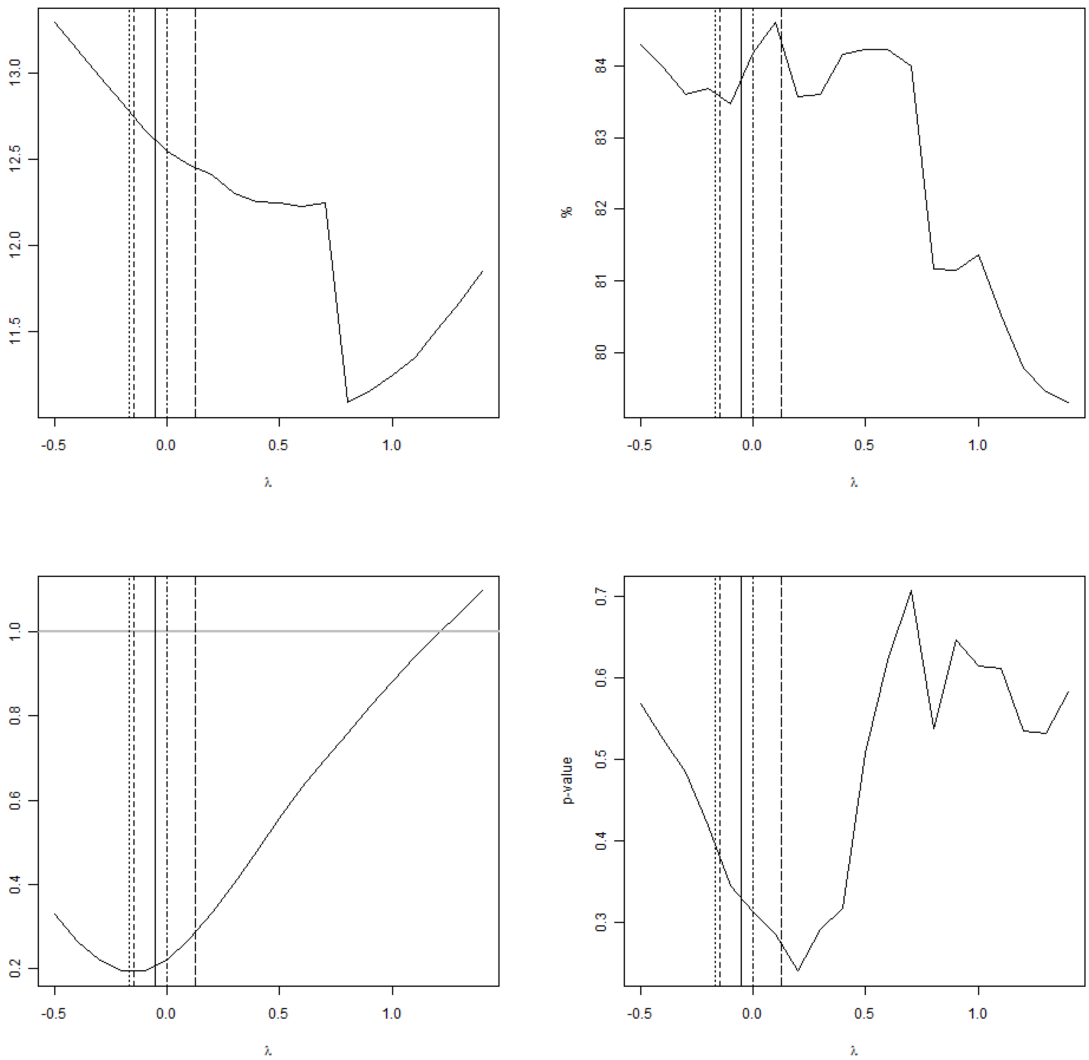
With respect to the multiplicative adjustment, $\hat{\lambda}_G = -0.17$ was shown to be the best performer out of the other three estimators, in terms of achieving reductions of 8.5 – 20.1% in revisions to seasonally adjusted and trend level and movement estimates. Improvement in revision performance compared to the multiplicative transform was also observed for $\hat{\lambda}_{M7} = -0.15$, followed by $\hat{\lambda}_D = -0.1$. Interestingly, the maximum likelihood estimator yielded a positive value of λ, and did not produce any gains with respect to the multiplicative transform (see table 5.31).

The residual seasonality plot below suggests that there is no strong evidence of residual seasonality in the seasonally adjusted estimates under all investigated λ (see figure 5.30).

5.29 OAD – Departures to France



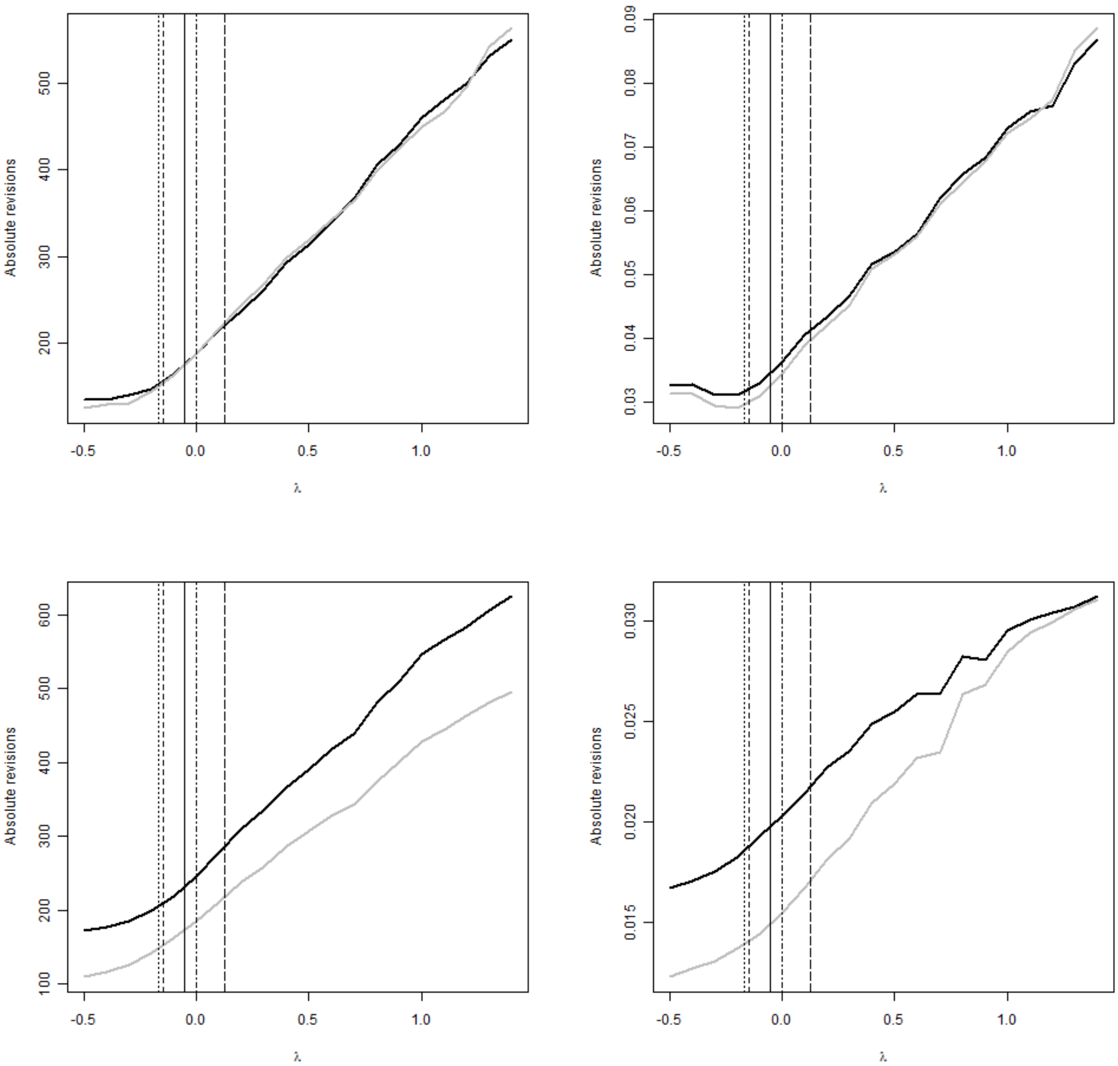
5.30 OAD – Departures to France, Quality Measures against lambda values
(from L–R, T–B, AAPC, RCVG, M7 and residual seasonality p-value: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))



5.31 OAD – Departures to France, Percentage performance of estimators with respect to multiplicative adjustment

	Estimator			
Quality Measure	*MLE (0.13)*	*Guerrero (-0.17)*	*Dummy (-0.05)*	*M7 (-0.15)*
AAPC	**0.7**	-1.9	-0.5	-1.6
RCVG	-0.4	**1.4**	**0.7**	**1.1**
M7	-28.4	**13.1**	**7.7**	**13.5**
Revisions to SA level lag 0	-18.5	**20.1**	**7.0**	**17.9**
Revisions to SA movement lag 0	-13.8	**17.3**	**5.0**	**14.1**
Revisions to T level lag 0	-17.6	**16.9**	**6.2**	**14.9**
Revisions to T movement lag 0	-7.9	**8.5**	**2.4**	**7.3**

5.32 OAD – Departures to France,
Lag 0 (black) and lag 1 (grey) Revision Measures against lambda values
(from L–R, T–B, seasonally adjusted level, seasonally adjusted movement, trend level and trend movement estimates: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))



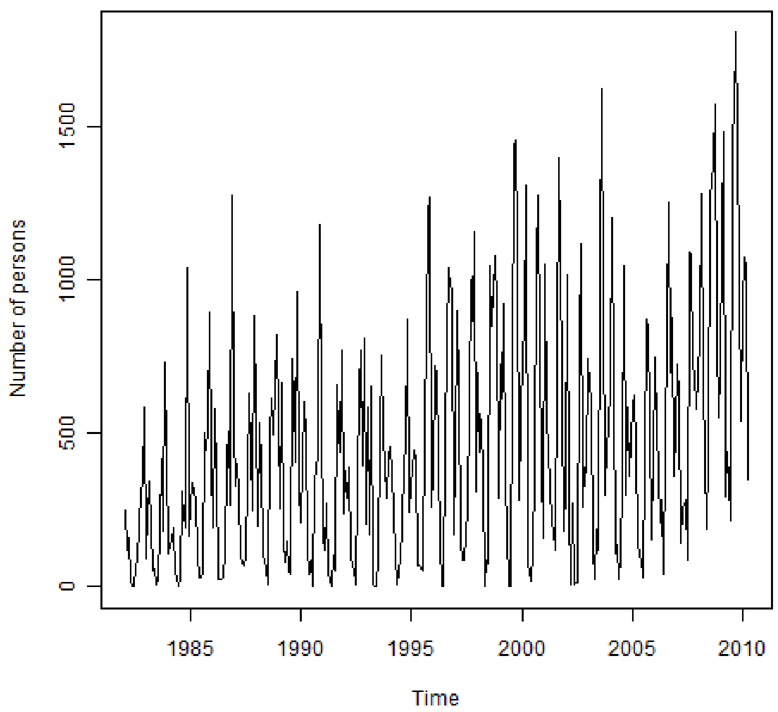
OAD – Departures to Nepal

As the plot of the original series shows, this series is rather volatile, with some visual evidence for moving seasonality. A few issues were encountered in the analysis of the series, the main one being that for some λ, either the seasonally adjusted estimates yielded extremely large values, or that some could not be computed (see Appendix D.1). Such results may obscure the computation and hence, interpretation, of our quality measures. Thus for this particular series, we restricted the study of the quality measures to be for the range $-0.2 \le \hat{\lambda} \le 0.6$.

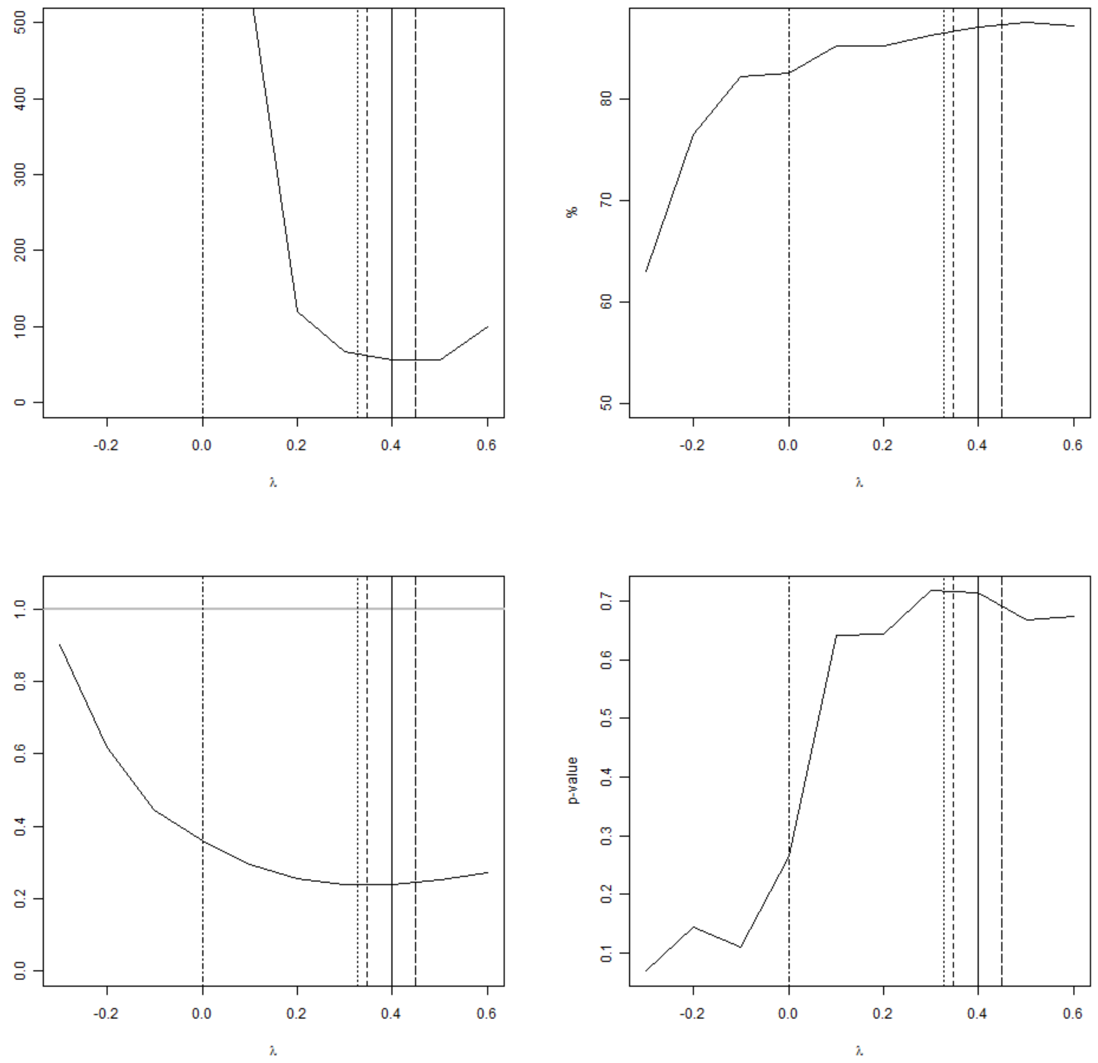
Revision plots for this series indicate that transforms in the interval $0.4 \le \hat{\lambda} \le 0.5$ can lead to improved seasonal adjustment (figure 5.36). In particular, the grid search found $\hat{\lambda} = 0.5$ to yield 11.5–39.1% reductions in revisions when compared with the multiplicative transform (Appendix C.5(b)).

Considering the four investigated methods, it can be seen from table 5.35 that all estimators outperform the current multiplicative option set for this series (except for the RCVG measure). The best performing estimator $\hat{\lambda}_{MLE} = 0.45$ yielded 11.3–41.2% reduction in revisions to seasonally adjusted and trend level and movement estimates with respect to the multiplicative transform. This was followed by $\hat{\lambda}_D = 0.4$, $\hat{\lambda}_{M7} = 0.35$ and lastly $\hat{\lambda}_G = 0.33$.

5.33 OAD – Departures to Nepal



5.34 OAD – Departures to Nepal, Quality Measures against lambda values
(from L–R, T–B, AAPC, RCVG, M7 and residual seasonality p-value: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))

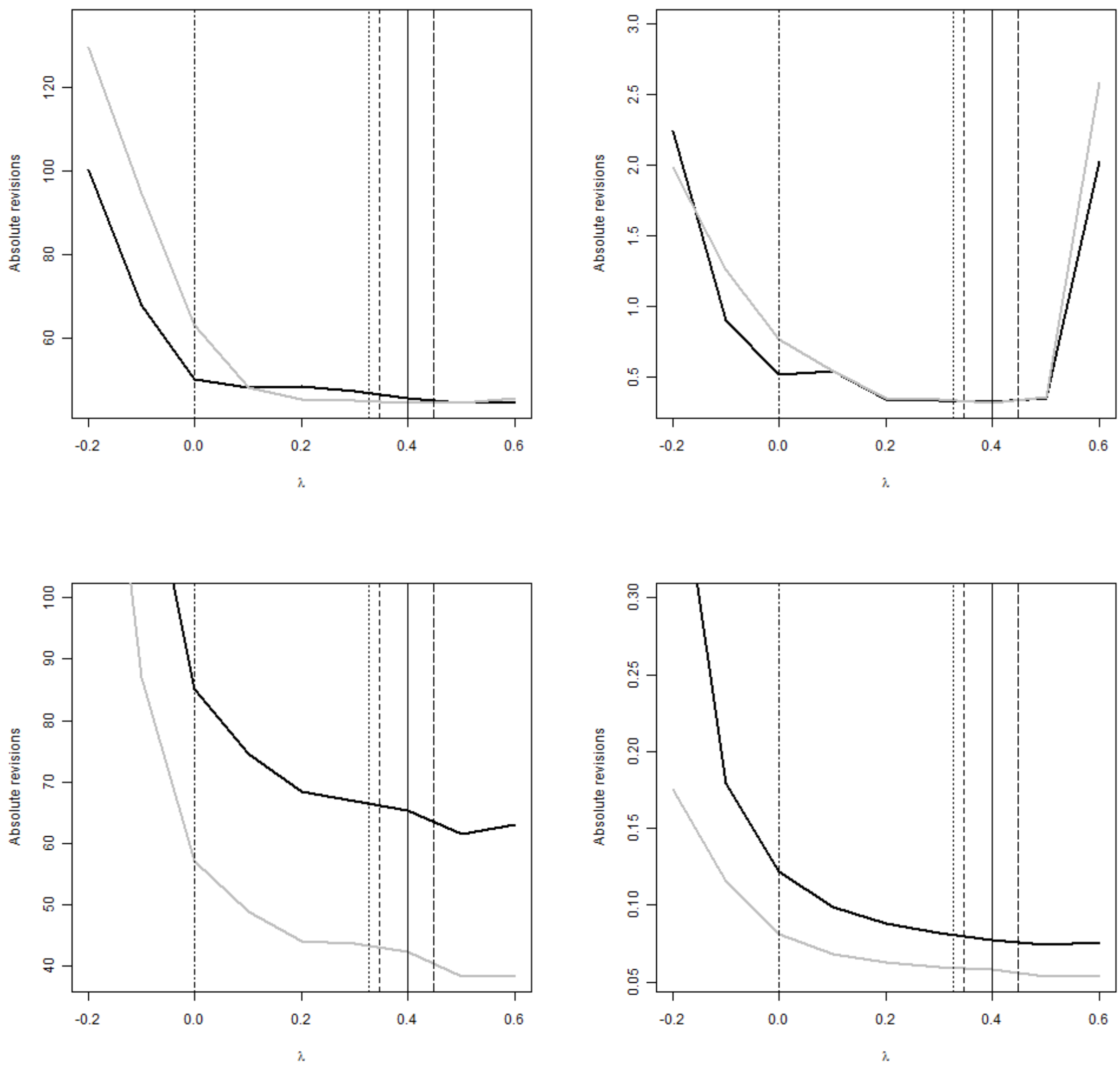


Seasonally low months April to July in this series has minimal activity but with some non-systematic exceptions, in particular July 1999, which reported zero departures. (see Appendix D.2). With the application of an inappropriate Box–Cox transform, such as $\lambda = -0.4$, the X11 filter-based method can potentially suffer from such large deviations, giving rise to singularities or extremely large or low estimates which are not feasible at the stage of reverse-transformation. Applying some sort of a bias correction may help solve this problem, though this was beyond the scope of our study.

5.35 OAD – Departures to Nepal, Percentage performance of estimators with respect to multiplicative adjustment

	Estimator			
Quality Measure	*MLE (0.45)*	*Guerrero (0.33)*	*Dummy (0.40)*	*M7 (0.35)*
AAPC	**97.8**	**97.5**	**97.7**	**97.6**
RCVG	-5.4	-5.1	-5.3	-5.2
M7	**31.8**	**33.8**	**33.2**	**33.8**
Revisions to SA level lag 0	**11.3**	**4.9**	**5.5**	**7.4**
Revisions to SA movement lag 0	**41.2**	**35.1**	**39.3**	**36.2**
Revisions to T level lag 0	**26.7**	**22.2**	**23.4**	**22.9**
Revisions to T movement lag 0	**39.2**	**33.5**	**36.1**	**34.6**

5.36 OAD – Departures to Nepal,
Lag 0 (black) and lag 1 (grey) Revision Measures against lambda values
(from L–R, T–B, seasonally adjusted level, seasonally adjusted movement, trend level and trend movement estimates: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))



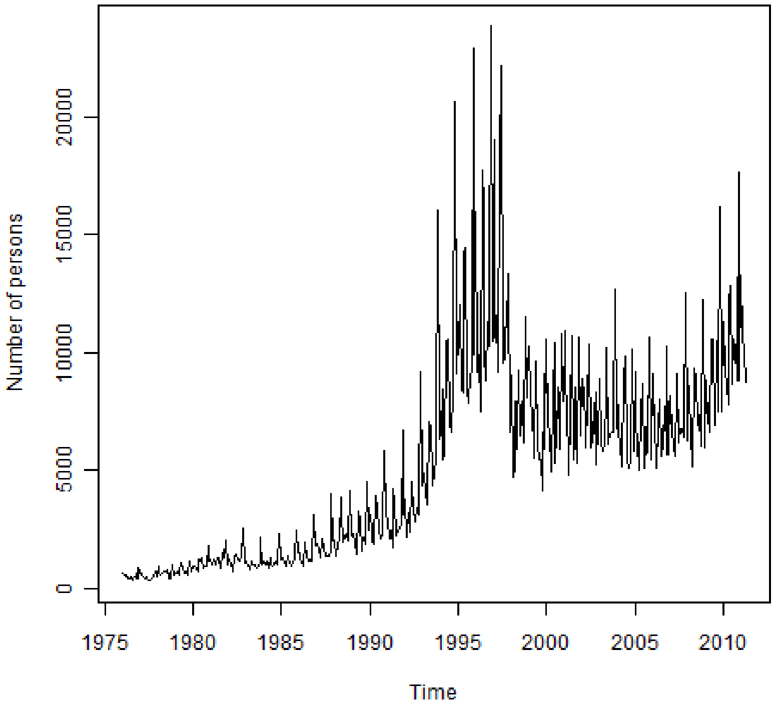
OAD – Arrivals from Indonesia

The seasonality in this series is very unusual. There looks to be a structural break around the time of the Indonesian currency crisis. This may explain the contradictory quality measures, that is, $\hat{\lambda} = -0.5$ being optimal for most revision measures but resulting in a greater evidence of residual seasonality surrounding the break (see figure 5.38). Note that the residual seasonality test did not present significant p-values as it is designed based on the last 5 years of data.

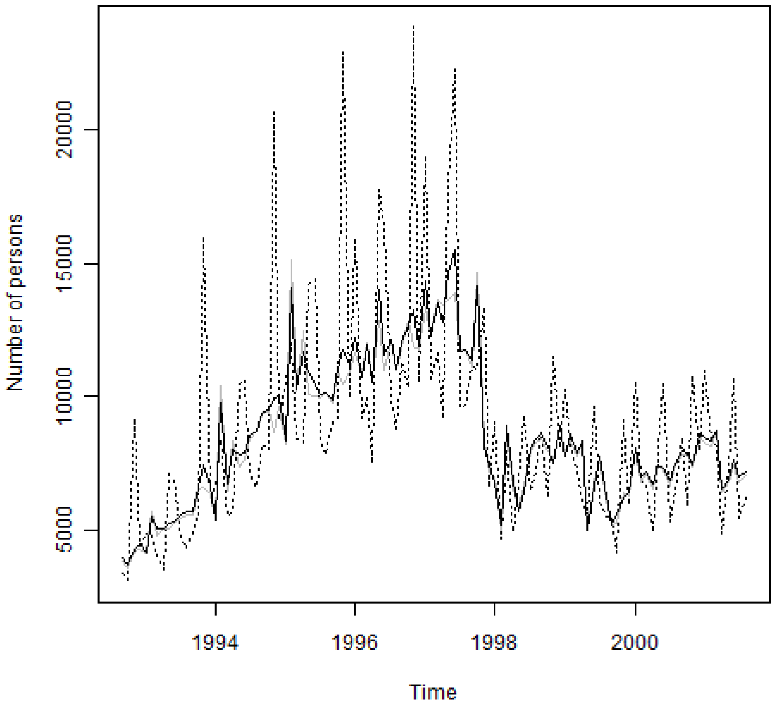
Like with the OAD – Departures to Nepal series, some transforms resulted in erroneous seasonal adjustment and so the study of the quality measures has been shortened to the range $-0.5 \le \hat{\lambda} \le 1.2$. The investigation found that the transform $\hat{\lambda} = -0.5$ resulted in 2.9–7.5% reductions across all revision measures when compared to the multiplicative transform (Appendix C.6(b)). In hindsight, it would be worth investigating this series under larger negative values of λ to see if more optimal revision measures can be obtained.

It is interesting to note that none of the four considered estimators reported values close to –0.5. In terms of reducing revisions to trend level and movement estimates, $\hat{\lambda}_{M7} = -0.07$ performed the best with respect to the multiplicative transform, followed shortly by $\hat{\lambda}_G = -0.04$ and $\hat{\lambda}_D = -0.03$ (see table 5.40). These gains are not great, considering the 2.4 – 5.8% offsets in revisions to seasonally adjusted level and movement estimates. On another note, $\hat{\lambda}_{MLE}$ reported a positive value of λ and was found to present no substantial gains over the multiplicative adjustment.

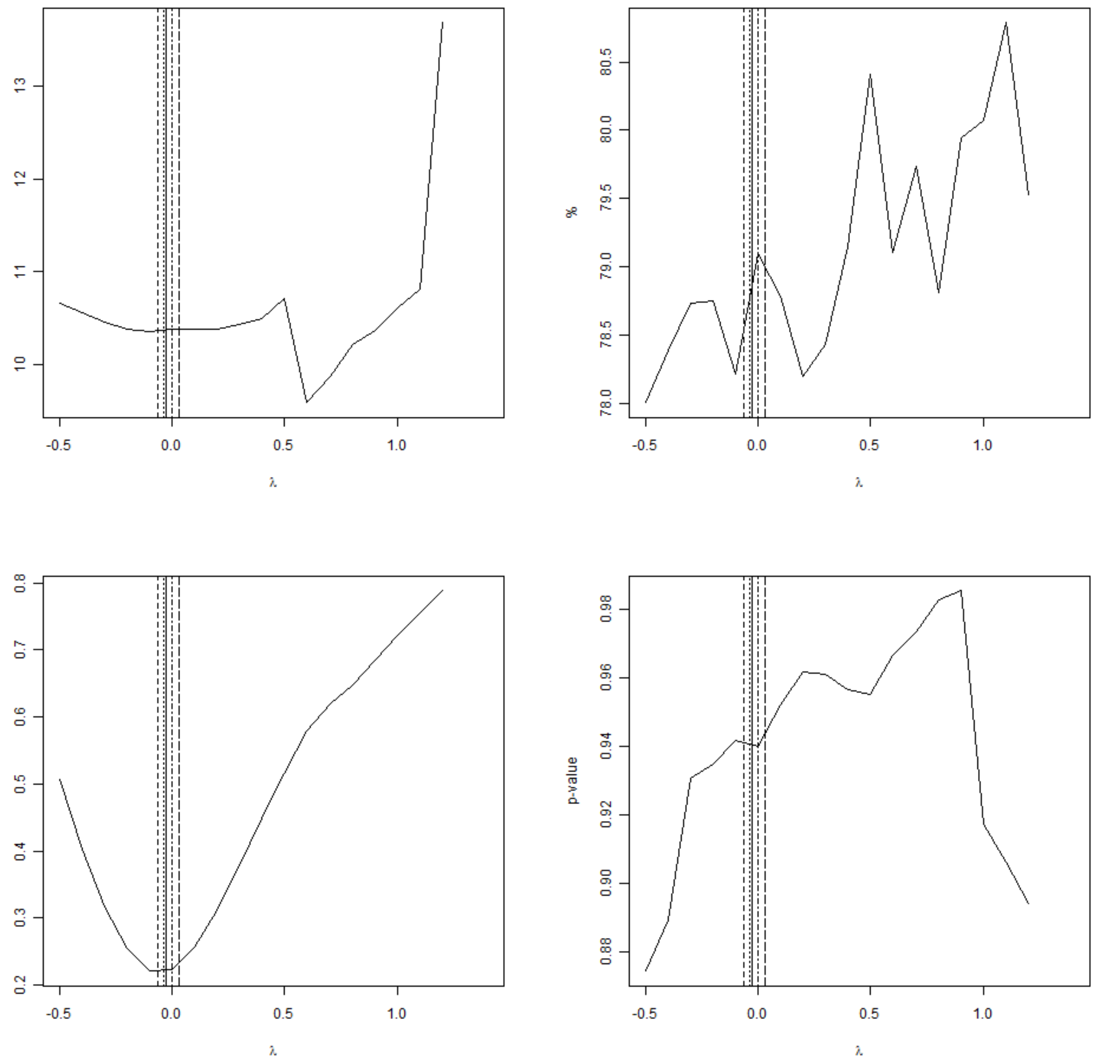
5.37 OAD – Arrivals from Indonesia



5.38 Residual seasonality in Septembers – Indonesia currency crisis



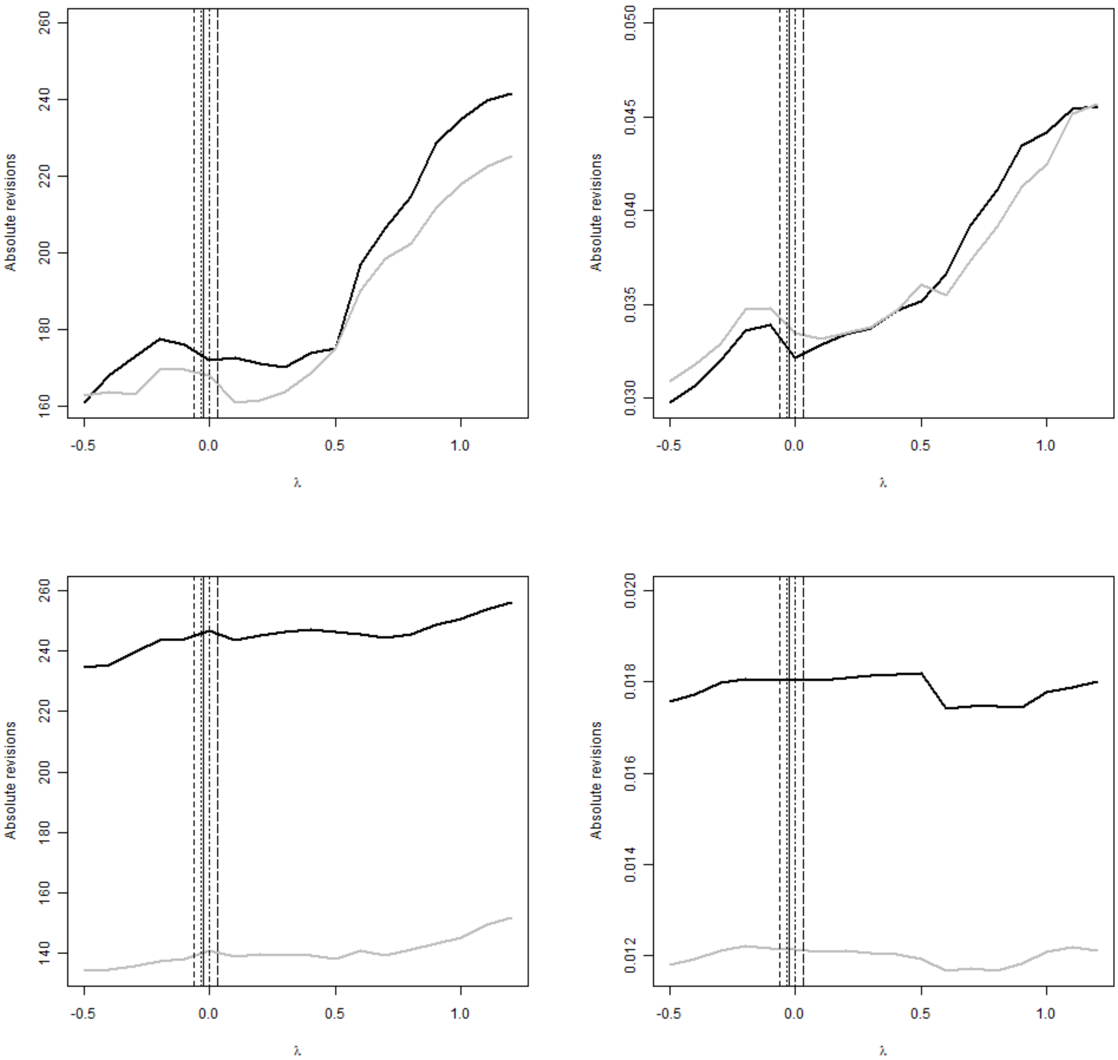
5.39 OAD – Arrivals from Indonesia, Quality Measures against lambda values
(from L–R, T–B, AAPC, RCVG, M7 and residual seasonality p-value: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))



5.40 OAD – Arrivals from Indonesia, Percentage performance of estimators with respect to multiplicative adjustment

	Estimator			
Quality Measure	*MLE (0.03)*	*Guerrero (-0.04)*	*Dummy (-0.03)*	*M7 (-0.07)*
AAPC	0	**0.1**	**0.1**	**0.1**
RCVG	-0.1	**0.4**	**0.3**	**0.6**
M7	-3.6	**2.2**	**1.8**	**2.7**
Revisions to SA level lag 0	-0.5	-2.4	-2.8	-3
Revisions to SA movement lag 0	-0.7	-5.7	-5.7	-5.8
Revisions to T level lag 0	**0.2**	**0.4**	**0.2**	**1**
Revisions to T movement lag 0	0	**0.1**	-0.1	**0.2**

5.41 OAD – Arrivals from Indonesia,
Lag 0 (black) and lag 1 (grey) Revision Measures against lambda values
(from L–R, T–B, seasonally adjusted level, seasonally adjusted movement, trend level and trend movement estimates: MLE (long dash), Guerrero (dotted), M7 (short dash), Dummy (solid), Multiplicative (dot-dash))



6. CONCLUSIONS AND RECOMMENDATIONS

Users of official statistics can choose the original, seasonally adjusted or trend estimates either individually or in combination to aid in the decision making process. Where the use of seasonally adjusted estimates is appropriate, various methods and innovations have been investigated to see if the quality of seasonal adjustment can be improved. Two key aspects of a good quality seasonal adjustment are stable seasonal factors and minimal current end revisions. It has been shown that use of an appropriate Box–Cox transform in some cases can improve both of these.

This paper presents a study which evaluates four methods of selecting an appropriate Box–Cox parameter, namely a maximum likelihood approach, a time series variance stabilisation approach, a simple alternative approach based on seasonal dummy variables in a regression ARIMA model, and a method that optimises the $M7$ indicator. It is found for synthetically simulated data that the maximum likelihood method based on an ARIMA model specification gave the best results in recovering the true transformation parameter and also providing a transformation that allowed a high quality seasonal adjustment, compared to the other estimators.

Real data analysis showed particular series that can benefit from such a transformation for the purposes of seasonal adjustment. For series which benefited using a negative transformation, the $\hat{\lambda}_{MLE}$ estimator came out as the worst performer by not capturing the reciprocal nature of the time series. The $\hat{\lambda}_G$ estimator on the other hand performed best, however the values produced were still considerably far from the optimal $\hat{\lambda}$ achieved via the grid search. Where a square root transform was appropriate, the $\hat{\lambda}_{MLE}$ estimator produced the closest value that yielded the most optimal results.

Our study in this paper focuses only on monthly time series. Our study could be enhanced and broadened by (1) exploring bias corrections for the naïve estimators of the seasonally adjusted on the original scale, (2) extending simulation study to unveil any effect of trend and seasonal behaviour on the estimation of λ, (3) examining the effects of the Box–Cox approach on a contemporaneous aggregation structure including a high level aggregate time series, (4) explore prior correction of series in the context of Box–Cox transformations.

This has been an initial study aimed primarily at quantifying the revision performance, and quality of adjustment obtained using the Box–Cox transform. Our results potentially encourage us to include the Box–Cox transform as a viable decomposition option, into the ABS seasonal adjustment process. This proposed methodology, in some cases, provides a new approach to treat highly volatile time series, and those where the standard additive, multiplicative options fail to achieve constant seasonality.

7. FUTURE WORK

So far in our paper, we have outlined two main areas which require further research into, that of bias reduction in reverse-transformation, and the appropriate use of the HEGY seasonality test in detecting moving residual seasonality.

As pointed out by Proietti and Riani (2007), analytical expressions for reverse-transform bias corrections are currently not available under X11 methodology. The solution to this problem is yet to be derived. It would be highly desirable, provided such a solution exists, to have it implemented so as to improve accuracy in our seasonally adjusted estimates, should Box–Cox transforms be included in our seasonal adjustment practice.

In addition, further work needs to be put into the construction of a conceptually sound test for moving residual seasonality. We mentioned the HEGY seasonal unit test in Section 4 as an alternative to the currently available moving seasonality test in X11, and once again, highlight its practical implementation an area of future investigation.

Interpretation of seasonal factors under Box–Cox transformation is one issue that has not been covered in this paper. With the standard additive and multiplicative adjustments, the seasonal balance constraints are rather straightforward, allowing intuitive interpretations of the seasonal factors (or the use of seasonal-irregular charts) to be made. The question to be raised here is whether or not there exists a conceptual interpretation of seasonal factors on the transformed scale.

Throughout our study, we have been using input time series data with the assumption that they have been cleaned of outliers and structural breaks. Other effects such as moving holidays and trading day have also been estimated and removed before carrying out Box–Cox transformation. Simultaneous estimation of such corrections in the context of series undergoing a Box–Cox transformation will need to be further investigated.

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Of course, any errors in or omissions from the paper remain those of the authors.

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APPENDIXES

A. COMPARISON OF METHODS – A SIMULATION STUDY

In this section, we present a short simulation study designed to compare the three aforementioned methods for the estimation of λ. The assessment in this study is performance of method in recovering the true value of λ.

We suggest the use of a particular choice of $\hat{\lambda}$ in cases where it has been determined that the Box–Cox transformation of the data is desirable. Additionally, our results suggest that under certain circumstances, such a transformation of the data may not be preferred if the loss due to the introduction of a further parameter into the estimation procedure is not outweighed by the prospective gain. However, further investigation into this issue needs to be considered at a later time.

A.1 Procedure

The simulation procedure involved generating data using the additive decomposition model (1) and applying the inverse Box–Cox transform given by

$$O_t^{-(\lambda)} = \begin{cases} \left(1+\lambda O_t\right)^{1/\lambda} & \lambda \neq 0\,, \\ \exp\left(O_t\right) & \lambda = 0\,. \end{cases}$$

Essentially, we reverse engineer our data from a model which guarantees that a Box–Cox transformation will permit an additive decomposition. That is, the model was constructed with known trend, seasonal and irregular components.

For each simulation trial, the trend component is randomly chosen from the family of functions:

$$T_t = A \sin\left(\frac{B\,t}{N} + C\right), \; t = 1, \dots, N$$

where, independently, the parameters are chosen such that $A \sim U(0,3)$, $B \sim U(\pi/4, \pi)$ and $C \sim U(0, 2\pi)$. This choice was made to ensure that our results were robust with respect to a range of trend functions that genuinely exhibit smooth changes in level over time. Note also that B was chosen such that the trend does not contain cycles indicative of monthly seasonal frequencies, and kept distinguished from the seasonal component as defined within the ABS (see cat. no 1346.0.55.001, An Introductory Course on Time Series Analysis).

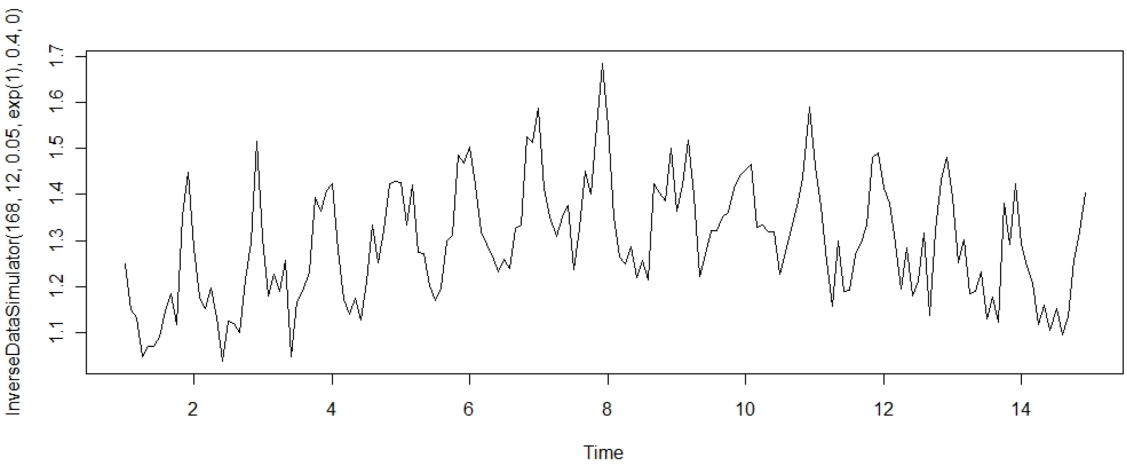
The seasonal components for the simulated monthly time series are given by

$$S_i = \frac{(i-6)^2}{144}, \; i = 1, \ldots, 12$$

and in some sense, resembled data that we might expect to see in real world data sets. Note that the sum of these components does not equate to 12, but this seasonal balance constraint can be easily satisfied by incorporating the appropriate constants into the trend component. Lastly, the noise or irregular components are error terms sampled from independent $N(0,\sigma^2)$ distributions.

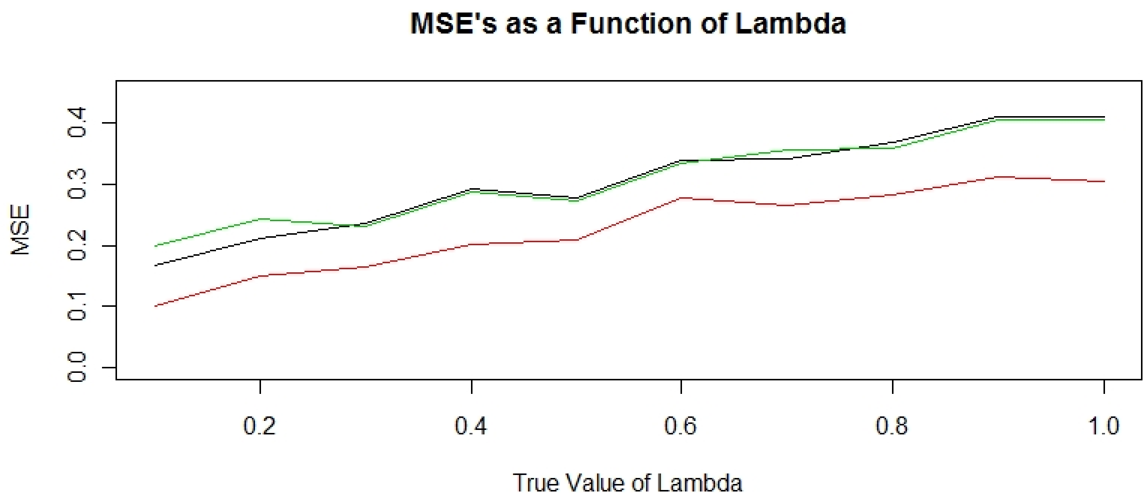
For every tenth increment of λ on the interval $(0,1]$, time series data was simulated 800 times according to the components defined above. Estimators of the known true λ were obtained using each of the three methods. Results were then collated for each increment and estimation method so that their performance could be compared against each other over a range of values of λ by means of the Mean Square Error (MSE). It should be noted however, that our particular calculation of the MSE is somewhat atypical. Since it was known that the true value of λ lies in the interval $(0,1]$, we truncated any estimates of λ outside of the interval $[-1,2]$ to lie on the appropriate boundary of the interval. The reason behind this particular choice was to ensure that any wildly inaccurate estimates were not incorporated into the MSE. Moreover, this particular choice has some credibility, since in practice, we are only likely to want to employ the Box–Cox transformation when the data indicates that the model lies somewhere between the multiplicative and additive model (inclusive), that is, $\lambda \in [0,1]$.

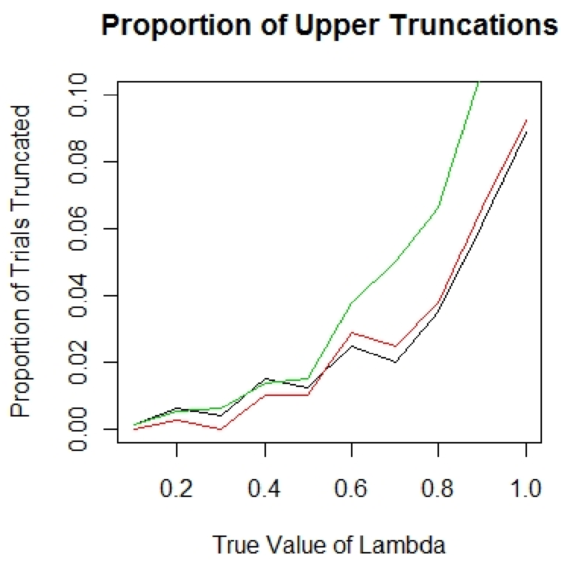
We provide, as an example, the figure below as a 'typical' data set that might be generated as a result of applying the inverse Box–Cox transform to the additive model as defined by the above components. In this case, the true value for λ was chosen to be 0.4.

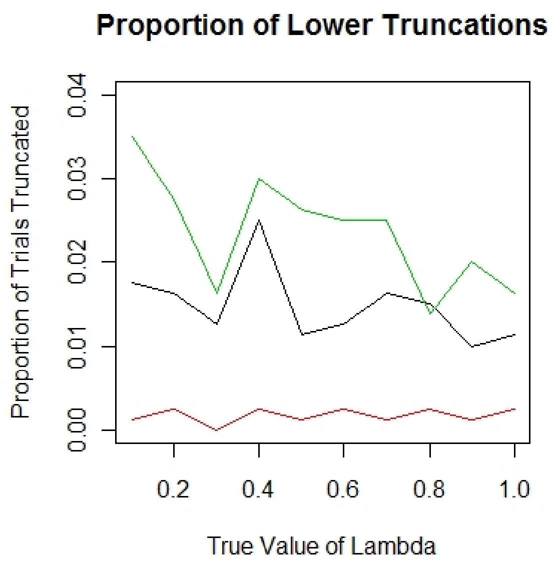


A.2 Results

The results for the MSEs are displayed graphically in the figures below. Additionally, we present results which report on the number of truncations for each of the estimation procedures. The black, red and green lines represent the methods of Guerrero, MLE and dummy-variable regression respectively.





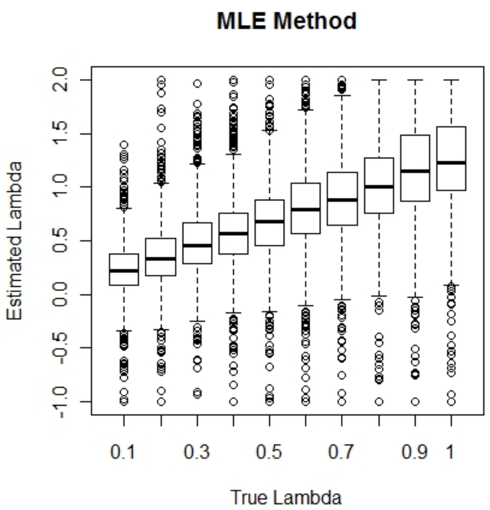


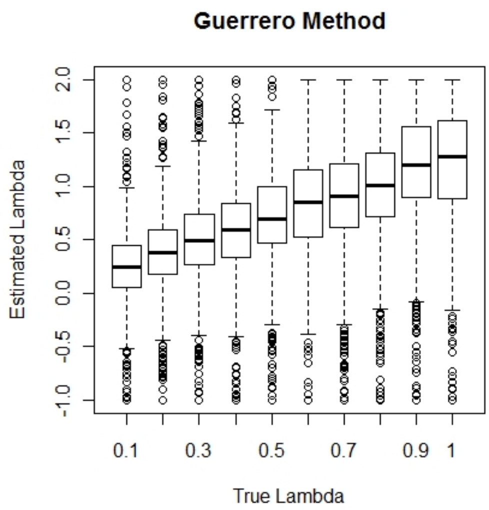
The most evident point that should be observed from an analysis of the simulation study is in regards to the performance of $\hat{\lambda}_{MLE}$. This method of maximum likelihood clearly outperformed both alternative methods over the entire range of possible λ that were considered. Furthermore, it should be noted that this method also involved the least number of truncations that needed to be performed.

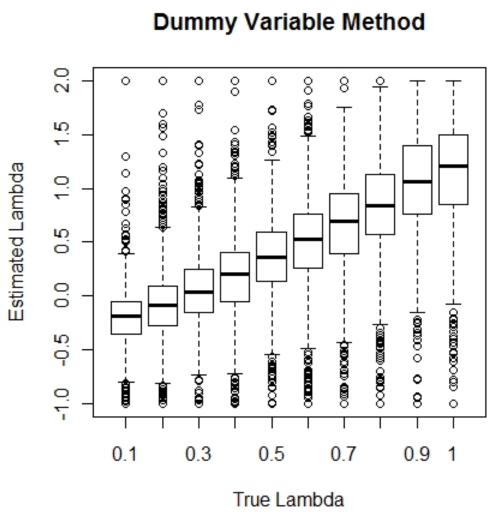
It may be noted that when the MSE of the estimates of λ are considered as an appropriate measure of the performance of those estimates, their performance worsens as λ increases. However, if we had considered the Relative Mean Square

Error as an appropriate indicator of performance, the situation would be reversed and in this situation, the performance of all estimators increases as λ increases on the interval $(0,1]$. Notwithstanding, that the MLE estimator dominates the performance of the other two estimators is a position that remains unchanged for all reasonable indicators of performance.

A point that can be made here but is not observable from the figures presented above is that generally, both MLE and Guerrero estimation procedures can introduce a bias by overestimating λ. It would be simple to construct a density estimate of the distribution for each of $\hat{\lambda}_{MLE}$, $\hat{\lambda}_G$ and $\hat{\lambda}_D$ and even correct for the biases they introduce, but the value of doing so in this limited simulation study is questionable and beyond the scope of this paper. Nevertheless, we do present below the box and whisker plots for each of the estimation procedures. In particular, it can be observed that both the MLE and Guerrero methods produce an estimation such that $E\left(\hat{\lambda}\right) \approx \lambda + 0.25$.







B. RANKING OF SIMULATION RESULTS

B.1 Ranking of simulation results

	True lambda						
Quality measure / Estimator	*0*	*0.2*	*0.4*	*0.6*	*0.8*	*1*	*Sub-score*
MSE of SA							
MLE	1	2	3	3	2	1	12
Guerrero	3	1	2	2	3	3	14
M7	2	3	1	1	1	2	10
Dummy	4	4	4	4	4	4	24
MSE of T							
MLE	2	2	2	2	1	2	11
Guerrero	3	3	3	3	2	1	15
M7	1	1	1	1	3	3	10
Dummy	4	4	4	4	4	4	24
Revisions to SA level lag 0							
MLE	3	3	3	3	3	3	18
Guerrero	2	1	2	1	1	2	9
M7	1	2	1	2	2	1	9
Dummy	4	4	4	4	4	4	24
Revisions to SA movement lag 0							
MLE	3	3	3	3	3	3	18
Guerrero	1	2	2	1	1	2	9
M7	2	1	1	2	2	1	9
Dummy	4	4	4	4	4	4	24
Revisions to T level lag 0							
MLE	1	1	1	1	1	1	6
Guerrero	3	2	3	2	2	2	14
M7	2	3	2	3	3	3	16
Dummy	4	4	4	4	4	4	24
Revisions to T movement lag 0							
MLE	1	1	2	2	1	3	10
Guerrero	3	2	1	1	2	2	11
M7	2	3	3	3	3	1	15
Dummy	4	4	4	4	4	4	24
Revisions to SA level lag 1							
MLE	3	3	3	3	3	1	16
Guerrero	2	2	2	2	2	3	13
M7	1	1	1	1	1	2	7
Dummy	4	4	4	4	4	4	24
Revisions to SA movement lag 1							
MLE	3	3	3	3	3	1	16
Guerrero	1	2	2	1	1	2	9
M7	2	1	1	2	2	3	11
Dummy	4	4	4	4	4	4	24
Revisions to T level lag 1							
MLE	2	3	3	2	1	1	12
Guerrero	3	2	2	1	2	2	12
M7	1	1	1	3	3	3	12
Dummy	4	4	4	4	4	4	24
Revisions to T movement lag 1							
MLE	1	1	3	2	1	2	10
Guerrero	3	2	1	1	3	3	13
M7	2	3	2	3	2	1	13
Dummy	4	4	4	4	4	4	24
Total							
MLE	20	22	26	24	19	18	129
Guerrero	24	19	20	15	19	22	119
M7	16	19	14	21	22	20	112
Dummy	40	40	40	40	40	40	240

B.2 M7 performance relative to best results for each series

	True lambda					
	0	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
MLE	101.11	100.86	100.86	100.86	100.76	100.86
Guerrero	100.70	100.56	100.54	100.57	100.58	100.58
Dummy	**257.62**	**121.29**	**114.35**	**131.11**	**156.70**	**194.93**
M7	*100.62*	*100.00*	*100.00*	*100.03*	*100.04*	*100.00*
Best of 0, 1	*101.09*	*252.02*	*322.75*	*237.39*	*128.25*	*100.88*
True	*101.09*	*101.13*	*101.13*	*101.16*	*101.17*	*101.17*

C. SUMMARY OF REAL DATA ANALYSIS RESULTS

C.1(a) Quality Measures for Retail Trade, Australia – Total

λ	*AAPC*	*RCVG*	*M7*	*Residual seasonality p-values*
–0.5	0.7239	40.903	0.236	0.442940
–0.4	0.7181	40.887	0.200	0.458201
–0.3	0.7135	41.909	0.163	0.432404
–0.2	0.7122	42.504	0.129	0.442991
–0.1	0.7081	41.765	0.095	0.445455
0.0	0.7090	41.767	0.066	0.443126
0.1	0.7133	41.703	0.044	0.436932
0.2	0.7168	42.163	0.039	0.475087
0.3	0.7275	44.603	0.056	0.462394
0.4	0.7434	44.674	0.084	0.368444
0.5	0.7729	44.474	0.116	0.273629
0.6	0.7901	44.826	0.148	0.207870
0.7	0.8437	44.888	0.179	0.122296
0.8	0.8826	44.524	0.209	0.021207
0.9	0.9290	44.854	0.239	0.003541
1.0	0.9729	46.283	0.266	0.000005
1.1	1.0256	47.342	0.295	0.000001
1.2	1.0824	47.929	0.323	0.000000
1.3	1.1662	47.772	0.345	0.000000
1.4	1.2357	48.503	0.371	0.000000
$\hat{\lambda}_G = 0.22$	0.7191	42.862	0.041	0.470973
$\hat{\lambda}_{MLE} = 0.23$	0.7192	43.091	0.043	0.469106
$\hat{\lambda}_D = 0.25$	0.7202	43.789	0.045	0.467651
$\hat{\lambda}_{M7} = 0.17$	0.7158	41.579	0.038	0.479689

C.1(b) Quality (Revision) Measures for Retail Trade, Australia – Total

λ	*Seasonally adjusted level estimates Lag 0*	*Seasonally adjusted movement estimates Lag 0*	*Trend level estimates Lag 0*	*Trend movement estimates Lag 0*	*Seasonally adjusted level estimates Lag 1*	*Seasonally adjusted movement estimates Lag 1*	*Trend level estimates Lag 1*	*Trend movement estimates Lag 1*
–0.5	67.86	0.00420	76.51	0.00188	60.82	0.00390	56.31	0.00151
–0.4	58.26	0.00349	68.87	0.00178	51.43	0.00319	46.55	0.00140
–0.3	51.30	0.00286	64.88	0.00170	43.65	0.00268	41.98	0.00132
–0.2	44.23	0.00247	61.05	0.00164	35.79	0.00215	37.74	0.00121
–0.1	39.73	0.00223	59.29	0.00160	31.14	0.00188	35.76	0.00114
0.0	37.84	0.00218	58.40	0.00158	29.99	0.00182	34.53	0.00111
0.1	37.94	0.00223	58.06	0.00160	28.40	0.00181	33.34	0.00110
0.2	37.35	0.00227	57.37	0.00163	29.22	0.00202	32.19	0.00109
0.3	40.81	0.00253	56.24	0.00164	32.80	0.00234	31.20	0.00109
0.4	43.51	0.00269	56.02	0.00166	35.57	0.00251	31.94	0.00108
0.5	48.18	0.00307	56.58	0.00171	40.50	0.00291	34.77	0.00110
0.6	51.68	0.00327	57.97	0.00177	45.09	0.00314	37.55	0.00114
0.7	55.84	0.00360	62.01	0.00184	50.95	0.00350	41.61	0.00122
0.8	59.64	0.00392	66.83	0.00195	55.79	0.00387	46.27	0.00132
0.9	66.17	0.00453	75.10	0.00210	63.23	0.00456	53.41	0.00150
1.0	76.44	0.00516	87.11	0.00229	78.66	0.00556	66.30	0.00175
1.1	91.67	0.00625	100.49	0.00249	91.14	0.00634	77.86	0.00202
1.2	109.91	0.00745	116.13	0.00269	108.62	0.00790	91.56	0.00227
1.3	114.04	0.00776	126.25	0.00289	112.75	0.00822	97.64	0.00239
1.4	138.87	0.00929	146.67	0.00325	131.90	0.00931	115.72	0.00277
$\hat{\lambda}_G = 0.22$	37.64	0.00232	57.16	0.00163	29.60	0.00208	31.85	0.00109
$\hat{\lambda}_{MLE} = 0.23$	38.07	0.00236	56.84	0.00163	30.09	0.00213	31.47	0.00109
$\hat{\lambda}_D = 0.25$	38.80	0.00241	56.70	0.00164	30.60	0.00214	31.32	0.00109
$\hat{\lambda}_{M7} = 0.17$	37.17	0.00220	57.47	0.00162	28.17	0.00191	32.53	0.00109

C.2(a) Quality Measures, Retail Trade, Western Australia – Total

λ	*AAPC*	*RCVG*	*M7*	*Residual seasonality p-values*
–0.5	1.18164	53.783	0.252	0.557469
–0.4	1.2476	56.894	0.211	0.562061
–0.3	1.2442	56.497	0.172	0.729722
–0.2	1.2364	56.689	0.134	0.807653
–0.1	1.2282	56.537	0.100	0.873891
0.0	1.2247	55.744	0.073	0.906762
0.1	1.2199	55.930	0.057	0.913849
0.2	1.2206	56.606	0.060	0.915901
0.3	1.2254	56.886	0.079	0.913651
0.4	1.2340	56.619	0.109	0.843219
0.5	1.1840	54.095	0.141	0.838269
0.6	1.1977	53.456	0.175	0.627502
0.7	1.2735	54.113	0.207	0.392173
0.8	1.3149	54.329	0.240	0.255675
0.9	1.3549	54.452	0.271	0.141312
1.0	1.3854	54.569	0.302	0.076368
1.1	1.4292	54.970	0.333	0.046563
1.2	1.5583	55.544	0.361	0.001368
1.3	1.6354	55.812	0.389	0.000284
1.4	1.7213	55.399	0.417	0.000070
$\hat{\lambda}_G = 0.18$	1.2204	56.540	0.057	0.915087
$\hat{\lambda}_{MLE} = 0.23$	1.2215	56.598	0.064	0.916376
$\hat{\lambda}_D = 0.21$	1.2209	56.596	0.061	0.916664
$\hat{\lambda}_{M7} = 0.12$	1.2196	56.358	0.056	0.914178

C.2(b) Quality (Revision) Measures, Retail Trade, Western Australia – Total

λ	*Seasonally adjusted level estimates Lag 0*	*Seasonally adjusted movement estimates Lag 0*	*Trend level estimates Lag 0*	*Trend movement estimates Lag 0*	*Seasonally adjusted level estimates Lag 1*	*Seasonally adjusted movement estimates Lag 1*	*Trend level estimates Lag 1*	*Trend movement estimates Lag 1*
–0.5	10.03	0.00482	12.29	0.00284	9.14	0.00510	8.43	0.00208
–0.4	9.00	0.00459	11.51	0.00278	8.27	0.00486	7.66	0.00195
–0.3	8.16	0.00422	10.83	0.00271	7.51	0.00448	6.98	0.00185
–0.2	7.33	0.00392	10.48	0.00271	6.81	0.00408	6.55	0.00181
–0.1	6.50	0.00371	10.34	0.00276	5.98	0.00375	6.19	0.00185
0.0	6.02	0.00369	10.02	0.00277	5.38	0.00357	5.78	0.00184
0.1	5.67	0.00364	9.73	0.00278	4.95	0.00342	5.53	0.00183
0.2	5.48	0.00391	9.65	0.00281	5.09	0.00384	5.30	0.00183
0.3	5.77	0.00429	9.46	0.00286	5.45	0.00424	5.05	0.00184
0.4	5.99	0.00450	9.38	0.00288	5.83	0.00462	4.93	0.00186
0.5	6.94	0.00491	9.50	0.00290	6.66	0.00515	5.17	0.00190
0.6	7.30	0.00527	9.59	0.00293	6.94	0.00548	5.45	0.00192
0.7	7.58	0.00553	9.76	0.00296	7.42	0.00594	5.65	0.00195
0.8	8.21	0.00611	10.04	0.00296	8.07	0.00642	5.90	0.00198
0.9	9.25	0.00702	10.74	0.00302	9.52	0.00757	6.71	0.00211
1.0	10.51	0.00775	11.95	0.00316	10.45	0.00831	7.77	0.00231
1.1	11.76	0.00857	12.95	0.00334	11.51	0.00914	8.76	0.00251
1.2	12.88	0.00924	14.09	0.00353	12.81	0.00997	9.83	0.00272
1.3	14.73	0.01081	15.31	0.00375	14.77	0.01115	10.90	0.00294
1.4	16.08	0.01166	16.72	0.00394	16.53	0.01224	12.11	0.00312
$\hat{\lambda}_G = 0.18$	5.49	0.00387	9.69	0.00281	5.06	0.00378	5.37	0.00184
$\hat{\lambda}_{MLE} = 0.23$	5.49	0.00403	9.55	0.00282	5.15	0.00393	5.18	0.00183
$\hat{\lambda}_D = 0.21$	5.51	0.00396	9.62	0.00281	5.09	0.00387	5.26	0.00183
$\hat{\lambda}_{M7} = 0.12$	5.65	0.00370	9.70	0.00278	4.98	0.00348	5.49	0.00183

C.3(a) Quality Measures, Retail Trade, Australia – Newspaper and Book Retailing

λ	*AAPC*	*RCVG*	*M7*	*Residual seasonality p-values*
–0.5	1.6895	66.965	0.172	0.99546
–0.4	1.6829	66.064	0.157	0.99578
–0.3	1.6798	66.538	0.144	0.99652
–0.2	1.6795	66.043	0.133	0.99695
–0.1	1.6781	65.059	0.126	0.99732
0.0	1.6820	66.134	0.121	0.99740
0.1	1.6846	66.964	0.119	0.99754
0.2	1.6923	67.036	0.120	0.99789
0.3	1.7023	66.616	0.123	0.99759
0.4	1.7124	66.858	0.128	0.99819
0.5	1.7190	66.832	0.135	0.99834
0.6	1.7227	66.329	0.143	0.99852
0.7	1.7294	66.214	0.152	0.99881
0.8	1.7386	66.628	0.162	0.99890
0.9	1.7532	66.866	0.173	0.99896
1.0	1.7731	68.433	0.184	0.99906
1.1	1.7979	69.417	0.196	0.99917
1.2	1.8266	69.722	0.207	0.99933
1.3	1.8525	68.130	0.219	0.99941
1.4	1.8853	68.529	0.230	0.99980
$\hat{\lambda}_G = -0.09$	1.6782	65.118	0.125	0.99733
$\hat{\lambda}_{MLE} = 0.01$	1.6824	65.839	0.121	0.99742
$\hat{\lambda}_D = -0.07$	1.6782	65.261	0.124	0.99727
$\hat{\lambda}_{M7} = 0.08$	1.6838	66.950	0.119	0.99748

C.3(b) Quality (Revision) Measures, Retail Trade, Australia – Newspaper and Book Retailing

λ	*Seasonally adjusted level estimates Lag 0*	*Seasonally adjusted movement estimates Lag 0*	*Trend level estimates Lag 0*	*Trend movement estimates Lag 0*	*Seasonally adjusted level estimates Lag 1*	*Seasonally adjusted movement estimates Lag 1*	*Trend level estimates Lag 1*	*Trend movement estimates Lag 1*
–0.5	2.70	0.00707	3.59	0.00446	2.45	0.00717	2.29	0.00329
–0.4	2.70	0.00703	3.57	0.00443	2.42	0.00708	2.26	0.00327
–0.3	2.69	0.00715	3.56	0.00442	2.41	0.00707	2.26	0.00325
–0.2	2.67	0.00729	3.55	0.00440	2.42	0.00720	2.26	0.00324
–0.1	2.66	0.00738	3.55	0.00439	2.41	0.00735	2.27	0.00324
0.0	2.70	0.00753	3.58	0.00440	2.47	0.00755	2.30	0.00328
0.1	2.73	0.00775	3.61	0.00440	2.52	0.00758	2.33	0.00332
0.2	2.80	0.00790	3.62	0.00439	2.53	0.00766	2.35	0.00332
0.3	2.78	0.00806	3.61	0.00439	2.61	0.00796	2.35	0.00331
0.4	2.78	0.00804	3.64	0.00438	2.63	0.00801	2.38	0.00334
0.5	2.83	0.00809	3.70	0.00443	2.68	0.00810	2.47	0.00340
0.6	2.91	0.00827	3.83	0.00457	2.76	0.00843	2.59	0.00351
0.7	3.03	0.00864	4.00	0.00469	2.86	0.00891	2.75	0.00362
0.8	3.22	0.00920	4.18	0.00483	3.03	0.00924	2.91	0.00376
0.9	3.31	0.00949	4.25	0.00488	3.10	0.00949	3.00	0.00380
1.0	3.45	0.00981	4.36	0.00494	3.18	0.00973	3.13	0.00388
1.1	3.52	0.01007	4.48	0.00501	3.29	0.01001	3.24	0.00395
1.2	3.63	0.01052	4.62	0.00509	3.49	0.01047	3.37	0.00405
1.3	3.84	0.01151	4.84	0.00521	3.84	0.01130	3.59	0.00420
1.4	3.89	0.01120	4.88	0.00530	3.87	0.01158	3.65	0.00423
$\hat{\lambda}_G = -0.09$	2.67	0.00740	3.56	0.00439	2.42	0.00737	2.27	0.00324
$\hat{\lambda}_{MLE} = 0.01$	2.70	0.00755	3.58	0.00440	2.47	0.00756	2.30	0.00328
$\hat{\lambda}_D = -0.07$	2.67	0.00739	3.57	0.00439	2.41	0.00740	2.27	0.00325
$\hat{\lambda}_{M7} = 0.08$	2.74	0.00772	3.60	0.00441	2.52	0.00754	2.32	0.00333

C.4(a) Quality Measures, OAD – Departures to France

λ	*AAPC*	*RCVG*	*M7*	*Residual seasonality p-values*
–0.5	13.2873	84.289	0.329	0.56843
–0.4	13.1320	83.984	0.267	0.52438
–0.3	12.9791	83.605	0.221	0.48391
–0.2	12.8250	83.681	0.195	0.41895
–0.1	12.6723	83.472	0.196	0.34497
0.0	12.5446	84.185	0.222	0.31207
0.1	12.4668	84.602	0.270	0.28494
0.2	12.4078	83.567	0.332	0.23971
0.3	12.2990	83.599	0.404	0.29014
0.4	12.2539	84.171	0.480	0.31642
0.5	12.2468	84.227	0.556	0.50757
0.6	12.2276	84.224	0.629	0.62108
0.7	12.2488	84.004	0.698	0.70642
0.8	11.0913	81.169	0.760	0.53670
0.9	11.1626	81.155	0.824	0.64676
1.0	11.2527	81.357	0.883	0.61430
1.1	11.3524	80.524	0.940	0.61081
1.2	11.5131	79.781	0.994	0.53452
1.3	11.6730	79.456	1.046	0.53167
1.4	11.8476	79.299	1.097	0.58242
$\hat{\lambda}_G = -0.17$	12.7869	83.024	0.193	0.38720
$\hat{\lambda}_{MLE} = 0.13$	12.4514	84.526	0.285	0.25826
$\hat{\lambda}_D = -0.05$	12.6076	83.604	0.205	0.32383
$\hat{\lambda}_{M7} = -0.15$	12.7445	83.244	0.192	0.37504

C.4(b) Quality (Revision) Measures, OAD – Departures to France

λ	*Seasonally adjusted level estimates Lag 0*	*Seasonally adjusted movement estimates Lag 0*	*Trend level estimates Lag 0*	*Trend movement estimates Lag 0*	*Seasonally adjusted level estimates Lag 1*	*Seasonally adjusted movement estimates Lag 1*	*Trend level estimates Lag 1*	*Trend movement estimates Lag 1*
–0.5	134.48	0.03259	171.69	0.01669	125.06	0.03139	109.35	0.01226
–0.4	135.47	0.03286	176.86	0.01702	129.68	0.03143	115.87	0.01271
–0.3	140.45	0.03110	185.75	0.01748	130.57	0.02946	125.02	0.01305
–0.2	146.76	0.03117	199.59	0.01821	144.58	0.02911	142.17	0.01371
–0.1	165.20	0.03302	218.26	0.01927	163.15	0.03096	162.28	0.01441
0.0	187.60	0.03631	246.04	0.02029	187.99	0.03459	185.23	0.01543
0.1	215.60	0.04056	277.82	0.02140	216.79	0.03895	209.88	0.01667
0.2	237.50	0.04346	310.18	0.02266	243.54	0.04200	237.50	0.01810
0.3	261.79	0.04674	335.56	0.02351	268.30	0.04524	258.56	0.01918
0.4	293.64	0.05175	365.58	0.02488	298.60	0.05094	285.74	0.02092
0.5	313.83	0.05346	390.58	0.02542	318.98	0.05308	306.61	0.02188
0.6	339.55	0.05629	416.77	0.02635	341.57	0.05590	327.12	0.02313
0.7	367.99	0.06200	439.56	0.02636	365.70	0.06108	344.11	0.02347
0.8	406.16	0.06570	481.68	0.02821	398.91	0.06447	373.77	0.02634
0.9	429.49	0.06831	510.89	0.02806	424.97	0.06782	401.77	0.02679
1.0	460.26	0.07294	546.47	0.02952	449.46	0.07210	429.40	0.02845
1.1	481.34	0.07558	565.90	0.03004	467.91	0.07443	445.10	0.02940
1.2	499.82	0.07649	584.68	0.03036	495.55	0.07741	463.32	0.02991
1.3	531.40	0.08301	606.25	0.03068	542.73	0.08506	481.10	0.03053
1.4	550.03	0.08681	625.15	0.03118	563.70	0.08855	496.27	0.03100
$\hat{\lambda}_G = -0.17$	149.96	0.03004	204.45	0.01857	148.85	0.02891	147.67	0.01392
$\hat{\lambda}_{MLE} = 0.13$	222.29	0.04134	289.26	0.02190	224.91	0.03940	219.25	0.01727
$\hat{\lambda}_D = -0.05$	174.52	0.03449	230.77	0.01981	174.05	0.03279	173.00	0.01486
$\hat{\lambda}_{M7} = -0.15$	154.04	0.03118	209.32	0.01880	153.28	0.02948	152.66	0.01411

C.5(a) Quality Measures, OAD – Departures to Nepal

λ	*AAPC*	*RCVG*	*M7*	*Residual seasonality p-values*
–0.5	9,412,887	50.123	1.862	0.53112
–0.4	NA	NA	NA	NA
–0.3	60,022.42	62.980	0.900	0.06932
–0.2	15,451.06	76.452	0.618	0.14352
–0.1	8,015.445	82.142	0.444	0.10951
0.0	2,490.098	82.522	0.358	0.26548
0.1	547.2618	85.130	0.294	0.64256
0.2	120.5355	85.170	0.255	0.64439
0.3	66.9848	86.233	0.238	0.71763
0.4	55.8123	87.063	0.239	0.71355
0.5	55.9908	87.440	0.252	0.66899
0.6	100.4155	87.099	0.273	0.67472
0.7	NA	NA	NA	NA
0.8	NA	NA	NA	NA
0.9	NA	NA	NA	NA
1.0	70.7526	86.603	0.408	0.31365
1.1	NA	NA	NA	NA
1.2	NA	NA	NA	NA
1.3	NA	NA	NA	NA
1.4	NA	NA	NA	NA
$\hat{\lambda}_G = 0.33$	62.4736	86.722	0.237	0.63553
$\hat{\lambda}_{MLE} = 0.45$	54.8836	87.002	0.244	0.76264
$\hat{\lambda}_D = 0.40$	56.3514	86.921	0.239	0.79337
$\hat{\lambda}_{M7} = 0.35$	60.1292	86.807	0.237	0.63877

C.5(b) Quality (Revision) Measures, OAD – Departures to Nepal

λ	*Seasonally adjusted level estimates Lag 0*	*Seasonally adjusted movement estimates Lag 0*	*Trend level estimates Lag 0*	*Trend movement estimates Lag 0*	*Seasonally adjusted level estimates Lag 1*	*Seasonally adjusted movement estimates Lag 1*	*Trend level estimates Lag 1*	*Trend movement estimates Lag 1*
–0.5	3,360.86	29.89409	35,248.91	17.87503	38,678.11	2,026.15	28,440.46	16.90483
–0.4	NA	NA	NA	NA	NA	NA	NA	NA
–0.3	NA	NA	NA	NA	NA	NA	NA	NA
–0.2	100.14	2.24133	213.18	0.41666	129.50	1.98909	160.49	0.17536
–0.1	67.94	0.90482	127.23	0.17895	94.62	1.26234	87.04	0.11586
0.0	50.34	0.51482	85.09	0.12176	63.08	0.76891	57.06	0.08135
0.1	48.21	0.54052	74.63	0.09865	48.37	0.54490	48.88	0.06807
0.2	48.51	0.34202	68.36	0.08804	45.41	0.35235	43.94	0.06249
0.3	47.47	0.33094	66.86	0.08211	45.29	0.34614	43.54	0.05967
0.4	45.79	0.32951	65.35	0.07746	44.55	0.31579	42.30	0.05803
0.5	44.57	0.34920	61.54	0.07418	44.71	0.35708	38.22	0.05350
0.6	44.85	2.02386	63.06	0.07583	45.69	2.59050	38.26	0.05340
0.7	NA	NA	NA	NA	NA	NA	NA	NA
0.8	NA	NA	NA	NA	NA	NA	NA	NA
0.9	NA	NA	NA	NA	NA	NA	NA	NA
1.0	53.53	1.67980	74.70	0.08774	55.36	1.66843	46.40	0.05864
1.1	NA	NA	NA	NA	NA	NA	NA	NA
1.2	NA	NA	NA	NA	NA	NA	NA	NA
1.3	NA	NA	NA	NA	NA	NA	NA	NA
1.4	NA	NA	NA	NA	NA	NA	NA	NA
$\hat{\lambda}_G = 0.33$	47.90	0.33412	66.16	0.08092	45.89	0.33898	42.51	0.05941
$\hat{\lambda}_{MLE} = 0.45$	44.65	0.30286	62.36	0.07400	44.50	0.30457	39.21	0.05477
$\hat{\lambda}_D = 0.40$	47.57	0.31240	65.18	0.07780	45.98	0.29669	41.74	0.05808
$\hat{\lambda}_{M7} = 0.35$	46.61	0.32854	65.61	0.07963	45.52	0.32595	42.39	0.05891

C.6(a) Quality Measures, OAD – Arrivals from Indonesia

λ	*AAPC*	*RCVG*	*M7*	*Residual seasonality p-values*
–0.5	10.6546	78.005	0.507	0.87460
–0.4	10.5530	78.385	0.403	0.88952
–0.3	10.4552	78.729	0.316	0.93087
–0.2	10.3825	78.750	0.254	0.93480
–0.1	10.3546	78.214	0.221	0.94188
0.0	10.3849	79.096	0.223	0.93994
0.1	10.3734	78.787	0.257	0.95216
0.2	10.3755	78.194	0.312	0.96174
0.3	10.4262	78.432	0.378	0.96091
0.4	10.4924	79.144	0.449	0.95659
0.5	10.7094	80.412	0.516	0.95505
0.6	9.5870	79.108	0.579	0.96639
0.7	9.8755	79.738	0.620	0.97345
0.8	10.2124	78.807	0.647	0.98279
0.9	10.3630	79.941	0.685	0.98531
1.0	10.6073	80.077	0.722	0.91749
1.1	10.8153	80.788	0.755	0.90632
1.2	13.6844	79.528	0.788	0.89416
1.3	NA	NA	NA	NA
1.4	NA	NA	NA	NA
$\hat{\lambda}_G$ = –0.04	10.3766	78.746	0.218	0.94731
$\hat{\lambda}_{MLE}$ = 0.03	10.3891	79.149	0.231	0.94348
$\hat{\lambda}_D$ = –0.03	10.3773	78.820	0.219	0.94817
$\hat{\lambda}_{M7}$ = –0.07	10.3733	78.653	0.217	0.94475

C.6(b) Quality (Revision) Measures, OAD – Arrivals from Indonesia

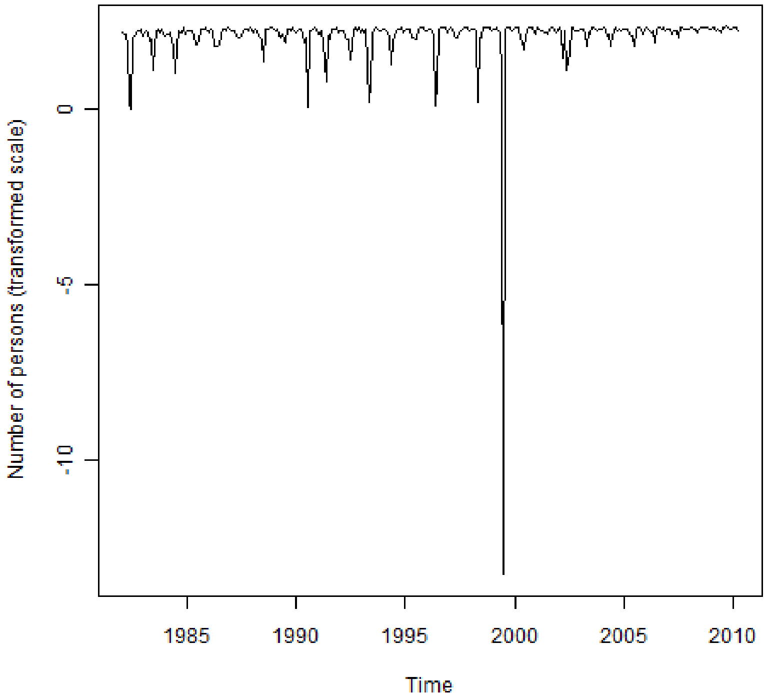
λ	*Seasonally adjusted level estimates Lag 0*	*Seasonally adjusted movement estimates Lag 0*	*Trend level estimates Lag 0*	*Trend movement estimates Lag 0*	*Seasonally adjusted level estimates Lag 1*	*Seasonally adjusted movement estimates Lag 1*	*Trend level estimates Lag 1*	*Trend movement estimates Lag 1*
–0.5	160.81	0.02974	234.87	0.01756	162.74	0.03090	133.89	0.01179
–0.4	168.10	0.03061	235.54	0.01773	163.53	0.03174	134.55	0.01191
–0.3	172.76	0.03201	239.61	0.01799	163.01	0.03291	135.38	0.01210
–0.2	177.56	0.03357	243.52	0.01806	169.37	0.03471	137.16	0.01220
–0.1	175.85	0.03392	244.16	0.01804	169.39	0.03477	137.73	0.01214
0.0	171.85	0.03216	246.77	0.01808	168.08	0.03346	140.66	0.01212
0.1	172.57	0.03285	243.80	0.01804	160.81	0.03317	138.56	0.01208
0.2	170.98	0.03342	245.41	0.01809	161.60	0.03346	139.28	0.01208
0.3	170.06	0.03370	246.29	0.01813	163.52	0.03380	139.07	0.01203
0.4	173.83	0.03469	247.37	0.01817	168.69	0.03469	139.15	0.01202
0.5	175.01	0.03520	246.55	0.01820	174.96	0.03608	138.02	0.01191
0.6	196.88	0.03660	245.57	0.01742	190.22	0.03551	140.79	0.01165
0.7	206.57	0.03928	244.62	0.01746	198.34	0.03735	139.18	0.01169
0.8	214.95	0.04108	245.75	0.01747	202.61	0.03913	141.16	0.01165
0.9	228.63	0.04346	248.60	0.01745	211.67	0.04127	143.00	0.01181
1.0	234.73	0.04418	250.60	0.01778	217.89	0.04250	145.05	0.01207
1.1	239.73	0.04541	254.01	0.01787	222.39	0.04518	149.25	0.01218
1.2	241.56	0.04557	256.37	0.01802	225.33	0.04567	151.57	0.01210
1.3	NA	NA	NA	NA	NA	NA	NA	NA
1.4	NA	NA	NA	NA	NA	NA	NA	NA
$\hat{\lambda}_G = –0.04$	175.92	0.03399	245.78	0.01806	169.78	0.03505	138.67	0.01214
$\hat{\lambda}_{MLE} = 0.03$	172.74	0.03239	246.25	0.01808	167.15	0.03348	140.03	0.01216
$\hat{\lambda}_D = –0.03$	176.66	0.03398	246.36	0.01809	170.56	0.03516	138.74	0.01217
$\hat{\lambda}_{M7} = –0.07$	176.99	0.03402	244.35	0.01805	169.77	0.03500	137.72	0.01216

D. ANALYSIS OF OAD – DEPARTURES TO NEPAL

D.1 Original series

	Jan	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sep*	*Oct*	*Nov*	*Dec*
1982	248.1	117.9	158.0	25.2	1.3	1.0	64.1	91.4	309.2	335.1	588.6	92.1
1983	236.5	343.4	194.0	51.1	56.5	4.6	25.3	179.5	415.9	178.7	734.7	107.4
1984	137.2	147.5	190.6	45.8	31.7	3.8	25.9	307.9	231.0	194.3	1,043.4	165.9
1985	266.2	340.1	282.7	107.7	31.6	29.4	42.5	500.4	443.5	518.4	894.0	402.8
1986	192.3	577.6	365.6	24.9	25.6	31.2	109.2	417.8	506.8	264.9	1,278.5	516.9
1987	326.5	398.1	305.7	98.3	82.2	70.4	93.2	629.6	449.7	251.3	883.3	573.6
1988	195.3	537.2	399.3	113.0	78.1	60.3	7.3	490.9	612.8	498.2	823.5	533.5
1989	251.6	662.1	142.7	78.9	149.1	52.4	38.3	744.0	610.1	405.1	963.5	321.7
1990	206.8	601.7	494.9	128.6	40.7	84.3	1.1	340.6	391.4	428.7	1,183.1	539.7
1991	163.3	118.3	271.1	44.7	2.5	101.3	53.1	658.6	497.7	439.8	772.8	235.8
1992	377.2	285.5	388.7	99.6	61.4	55.3	8.6	655.3	773.6	394.2	812.4	205.8
1993	586.7	167.7	651.4	6.0	1.2	2.3	94.8	485.2	753.4	497.7	289.9	423.7
1994	458.6	378.2	243.6	100.4	6.1	42.0	99.8	186.1	418.8	440.9	870.6	242.9
1995	329.0	447.3	418.7	69.2	71.1	63.7	52.6	533.6	820.5	1,219.0	1,273.1	261.3
1996	368.6	721.9	696.2	141.9	1.1	2.5	451.2	810.3	1,041.6	978.8	976.5	169.8
1997	413.5	902.9	650.0	150.1	83.0	86.0	254.1	599.7	1,001.2	867.0	1,156.9	308.1
1998	698.6	438.5	548.7	354.8	1.3	84.9	76.1	1,047.9	848.8	1,080.1	925.2	289.1
1999	728.0	614.9	922.1	352.3	174.2	3.8	0.0	671.2	1,439.2	1,457.5	1,127.8	281.9
2000	778.5	822.4	1,311.3	82.3	44.9	19.4	113.8	378.5	1,072.1	1,279.9	773.0	392.6
2001	158.6	1,052.6	558.4	324.8	213.9	178.5	121.2	716.6	1,404.1	1,129.7	611.2	180.6
2002	316.6	1,019.0	259.6	9.1	270.8	4.4	14.0	786.1	1,119.3	260.8	390.0	373.8
2003	745.2	665.1	609.8	118.3	23.6	184.6	118.7	1,028.6	1,624.3	296.4	475.1	498.2
2004	681.3	1,202.1	654.2	106.1	136.6	24.3	97.1	350.5	1,047.9	297.8	584.5	358.9
2005	597.8	626.5	450.3	150.5	98.2	90.7	26.8	405.4	874.6	844.5	558.1	152.4
2006	406.6	750.2	522.1	209.3	282.6	38.7	448.1	849.3	1,254.5	696.7	881.2	362.6
2007	511.3	724.1	618.2	143.1	251.5	282.6	85.4	1,094.4	1,085.9	716.4	703.7	578.5
2008	683.4	815.1	1,283.3	667.9	417.8	189.0	429.1	1,266.0	1,383.9	1,577.2	819.5	550.7
2009	697.1	1,153.5	1,486.2	296.0	442.1	342.5	214.0	1,457.0	1,556.8	1,809.7	1,482.6	540.1
2010	921.0	1,074.5	1,050.6	350.8								

D.2 Transformed original series $\lambda = -0.4$



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